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CONSIDERATION OF PHASE TRANSITION MECHANISMS DURING PRODUCTION IN MANUFACTURING PROCESSES

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Abstract. *In this study, by introducing the Ginzburg-Landau free energy, we define a parameter corresponding to an order parameter as a factor of the phase transition in manufacturing processes. Because thermal diffusion equations can be applied as mathematical models in the manufacturing process, we consider the applicability of the "Edge of Chaos", which is used in complex systems, to the manufacturing industry and the extent to which it would do so. We believe that in the manufacturing industry, the "Edge of Chaos" is a phenomenon that is caused by the loss of synchronization between the production and production throughput. The phase transition phenomenon is observed as the process throughput while manufacturing certain control equipment. We also verify the phase transition in the system through experiments on the flow production system. To maintain synchronization between the manufacturing and process throughput, it is necessary to know the critical point of the phase transition. From an economic perspective, it is important to focus on ways to prevent the critical point from being exceeded. In this study, we adopt the average value of the normalized rate-of-return deviations as the critical point. By not exceeding the average value of the rate-of-return, it is possible to maintain uninterrupted production.*
Keywords: Production density, Rate-of-return deviation, Phase transition, Potential energy, Ginzburg-Landau free energy

1. Introduction.

In recent years, much research related to phase transition theory has been reported. In particular, there has been significant progress in studies on physical phenomena.

In the field of statistical mechanics, the simulated annealing method is considered to be an efficient optimization technique. In this paper, we propose an approximation equation derived by the linearization of a fuzzy nonlinear membership function [1].

Optimization of both product manufacturing and transportation related to the manufacturing industry has been studied. The proposed technique is an optimized information system for automated guided vehicles and is not a centralized method [2].

In [3, 4], free energy is considered as the initial condition when mathematically treating a phase transition phenomenon. They noted that free energy does not increase in the case of a closed system. In contrast, it was reported that thermal noise must be considered when considering causative factors of phase transition.

Unlike the study by T. Tanabe and M. lshikawa, this paper attempts to analyze the phase transition mechanism in the manufacturing industry by treating manufacturing processes as a closed process when seen as a single manufacturing process, that is, a process on which external forces do not act. We instead define order parameters within a manufacturing process and further introduce the Ginzburg-Landau free energy.

In addition, we have reported that by creating a state in which the production density that of each process corresponds to the physical propagation, and the equation dominating the manufacturing process is indicated by a diffusion equation [5]. In the present paper, we analyze whether the "Edge of Chaos", which is discussed in complex systems, also exists in the manufacturing industry and the extent to which it does. We believe that in the manufacturing industry, the "Edge of Chaos" is caused by a loss of synchronization between the production and production throughput.

We indicate that the phase transition phenomenon is observed in the process throughput of the manufacture of certain control equipment. We verify the phase transition in the system through experiments on a flow production system.

To maintain synchronization between the production and production throughput, we need to know the critical point of the phase transition. From a business perspective, it is vital to focus on not exceeding the critical point. In the manufacturing industry, the critical point indicates the rate-of-return deviation. By not exceeding the average value of the rate-of-return, it is possible to maintain uninterrupted production. The reason for this is that the Ginzburg-Landau free energy can be treated as free energy for the manufacturing quantity between manufacturing processes. In other words, it can be considered the same as the throughput between manufacturing processes [6].

2. Mathematical Model of the Manufacturing Process.

We have reported that the analysis of the rate-of-return deviation for a certain equipment manufacturer over the past ten years has revealed "power-law distribution characteristics" [7]. Because the power-law distribution is a distribution revealing the existence of a phase transition phenomenon, we expect that there exists a correlation between the rate-of-return deviation and the production system in a manner that is mediated by the power-law distribution.

C. G. Langton is known for his 1990 study of artificial beings [8]. He also conceptualized the idea of the "Edge of Chaos". In physical phenomena, the "Edge of Chaos" refers to a phenomenon that corresponds to the transition state that exists between fluid and solid phases. Phenomena similar to the "Edge of Chaos" occur during the period from the entry of the manufacturing order for a product to its delivery. When an order for manufacturing is received, there exists an outflow of cash due to the purchase of materials for the person receiving the order, and there is a lead time until cash is injected at the end of the manufacturing period. To increase the rate-of-return, it is important to reduce the lead time from a financial perspective. In addition, the rate-of-return is decreased if opportunities are lost and if there are excessive inventory stocks. From a practical perspective, it is necessary to synchronize the speeds of individual manufacturing operations. We consider that the synchronization of manufacturing processes will lead to the improvement of the production throughput. Here, the synchronization of manufacturing processes is one method to enable the efficient progress of each process in order to increase the throughput.

By synchronizing the processes, each process will become more time efficient. In the under-production flow in Figure 1, because the production throughput is low relative to the production output, opportunity loss occurs (for example, a state where the shipping

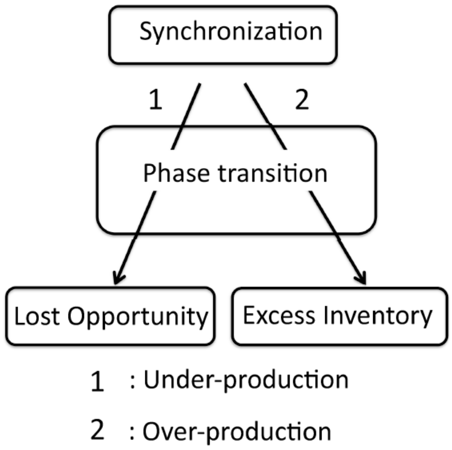

FIGURE 1. Lost opportunity (1: Under-production) and Excess inventory (2: Over-production)

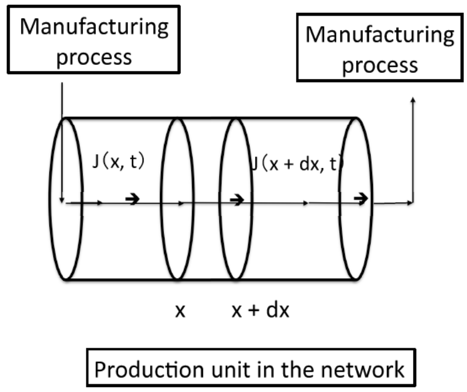

FIGURE 2. The diffusive propagation of products flowing through manufacturing process

quantity is small owing to endogenous and exogenous factors). In addition, in the over-production flow in Figure 1, because the throughput is very high relative to the production output, there will be excessive inventory. In any case, when such a probability structure exists, it is considered that a phase transition phenomenon occurs in the production field. Therefore, production management needs to be performed with this in mind.

From Figure 2, the present writers refer to a network capacity (static acceptable amount of production) in an inter-process network (a field of production) as R. An inter-process network means that, after one process is finished, a move to the next process is made, and, in such a manner, processes advance sequentially. Here, assuming that a production density function for the ith equipment is $S_i(x,t)$, $S_i(x,t)$ is expressed by

$$[J(x,t)dt - J(x+dx,t)dt]R = [S_i(x,t+dt) - S_i(x,t)]Rdx \tag{1}$$

where J is a production flow [5].

At this time, we define a production flow as displacement of a production density function in a unit production direction. In other words, a production density function is proportional to cost necessary for production, and thus it can be thought that it is production cost per unit production. Further, because performing production leads to obtaining a return, a production density function can be also considered as a return density function

$$\frac{\partial S_i(x,t)}{\partial t} = D\frac{\partial^2 S_i(x,t)}{\partial x^2} \tag{2}$$

where D is a diffusion coefficient. t is a time variable, and x is a spatial variable.

This equation is an equation of a form the same as a diffusion equation derived from a minimization condition of free energy in a production field [5]. It means that connection between processes can be treated as diffusive propagation of products (refer to Figure 2).

3. Potential Energy and Return.

Here, description that deviation of free energy produces a return will be made.

Assumption 1. *Return is created by liquidity of production density function $S_i(x,t)$. From this, there exists a potential that depends on a production density function.*

Here, the size of potential $F(S_i(x,t))$ is attributed to inclination of a production density function related to a production unit, that is, liquidity. Therefore, the following equation is

Definition 3.1.

$$\frac{dF(S_i(x,t))}{dS_i} = -\kappa \times grand\ S_i(x,t) \tag{3}$$

where κ is a constant.

In a manufacturing process that is a subject, attention is focused on a state change in $x \in [0, L]$, $t \in [0, T]$. That is, products are formed as a result of diffusive propagation for each production unit, and, after that, production progresses by iteration of that. Here, as a subject, it is supposed that behavior of an initial production density that is inputted from an opened production field at $t = 0$, the behavior being exhibited at $t \in [0, T]$ in a closed production field, is handled. In order to establish a model that constrains dynamic behavior of a production density for each production lead time, it is expressed as Equation (2) here.

Now, when variable transformation of $x = \theta + \rho L$, $t = \lambda$ is made, Equation (2) becomes

$$\frac{\partial S_i(x,t)}{\partial t} + \rho\frac{\partial S_i(x,t)}{\partial x} = D\frac{\partial^2 S_i(x,t)}{\partial x^2} \tag{4}$$

where D is a diffusion coefficient.

As above, Equation (4) indicates a diffusive field exhibiting advection (at transport velocity ρ), and is an equation constraining production density $S_i(x,t)$ in a field where a production unit is exhibiting advection at transport velocity ρ.

In other words, Figure 3 indicates that production elements are reduced due to change in a gathering form of production elements.

Then, eventually, production units become x_L ($x = L$) units, production elements become L, and a stream of a production density is ended. Thus, in $S_i(x,t)$, $x \in [0, L]$, $t \in [0, T]$, meaning of continuity of production unit x has been described, in particular. Therefore, it is considered that meaning of diffusive propagation such as Equation (4) has been made clear.

Next, the structure of potential in production density function $S_i(x,t)$ will be examined. Potential in the present research is defined as "ability to create a return".

By such definition, meaning of Equation (3) has been made clear. In other words, it is considered that inclination related to a production unit of potential $F(x,t)$ of production field $\{S_i(x,t)\}$ reduces in proportion to inclination related to a production unit of production density function $S_i(x,t)$, resulting in creating a return (it is considered as a difference between potentials).

When considering like this, we define potential energy (free energy) in a production field as follows.

Definition 3.2. *Potential energy in production field*

$$\begin{aligned} &[\textit{Potential of production field per production density}] \\ &= [\textit{Potential for production unit}] \\ &\quad + [\textit{Fluctuation of potential for production unit}] \end{aligned}$$

Such definition is almost equivalent to definition of the potential or free energy of a field in physics. We consider that a return is generated by temporal deviation of a potential function (free energy) attributed to a production density function. Therefore, let a cash flow function for each $t_k \in [0, T_k]$, $k = 0, 1, 2, \cdots$ be $P_k(t_p)$ (hereinafter, the subscript k is omitted).

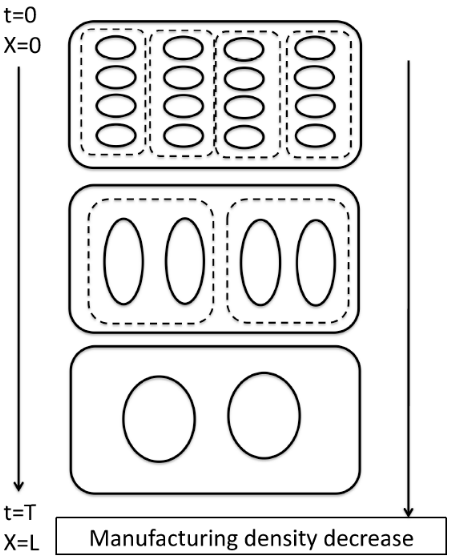

FIGURE 3. Production density decrease corresponding to manufacturing element integration

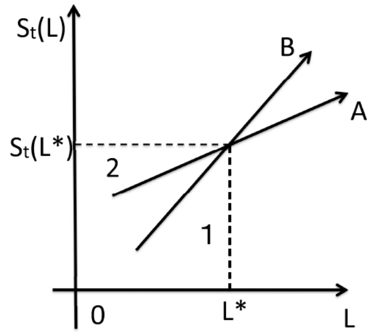

FIGURE 4. State variables for leadtime and throughput-time in manufacturing process

4. **Phase Transition in Manufacturing Industry.** Here, meaning of phase transition in the manufacturing industry will be described.

Now, as a state variable, lead time and throughput is considered. $S_t(L)$ of Figure 4 is defined as

Definition 4.1. *Production quantity $S_t(L)$ at time t, $x = L$*

$$S_t(L) = \int_0^L S(x,t)dx. \tag{5}$$

Production quantity $S_t(L^*)$ indicates a production quantity at the time when synchronization has been made, and L^* indicates lead time at the time when synchronization has been made. In addition, A and B in Figure 4 mean the following equations, respectively.

$$B : \left.\frac{dS_t(L)}{dL}\right|_{L\in 1} \equiv P_t^1(L) \tag{6}$$

$$A : \left.\frac{dS_t(L)}{dL}\right|_{L\in 2} \equiv P_t^2(L) \tag{7}$$

Therefore, let process throughput P be defined by

Definition 4.2.

$$P_t(L) = \frac{dS_t(L)}{dL}. \tag{8}$$

For this reason, when P is made to be changed with $L = constant$, it becomes as shown in Figure 5.

In Figure 5, P indicates process throughput, $F_t(P, L)$ is potential energy, and $P = P^*$ indicates that, at the phase transition point at the time when synchronization is made (synchronized throughput), production ability of a system becomes identical. When P is changed while L being fixed, $F_t^1(P, L)$ is a process with low production ability, and, when throughput is increased, potential is reduced rapidly. In contrast, $F_t^2(P, L)$ is a process with high production ability, and, even if throughput is raised, potential is not lowered

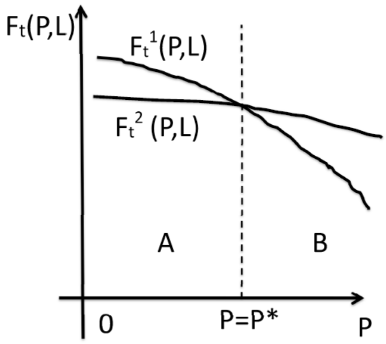

FIGURE 5. Potential energy is high or low when manufacturing power makes it high in process.

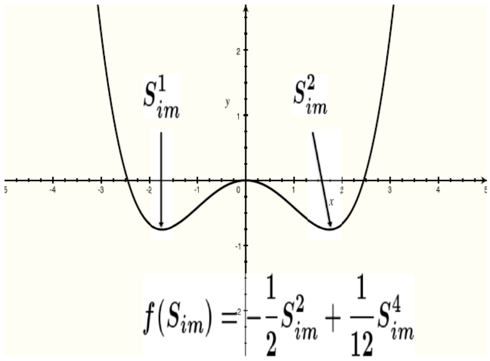

FIGURE 6. Two phases of non-production and production exist together

much. Here, the state of $F_t^1(P,L) = F_t^2(P,L)$ indicates a state that, at a so-called phase transition point $F_t^1(P,L) = F_t^2(P,L)$, throughput will be synchronized throughput (the throughput at the phase transition point) in the manufacturing industry. It means a state where production ability is identical.

Accordingly, as can be understood from Figure 5, according to expression of the statistical mechanics, an opportunity loss is easy to be caused when $P < P^*$, and excessive inventory is easy to be caused when $P > P^*$.

That is, in case of a system having multiple processes, when deviation from the synchronized throughput point is caused, production retention (production idle) is easy to occur between processes, and, as a whole system, this will create a state where excessive inventory (or, opportunity losses) is easy to occur.

In physics, in order to quantify order of such state, order parameter is introduced, free energy $F_t(P,L)$ is expressed as $F_t(P,L,S_i)$, and S_i is made to correspond to an order parameter.

Therefore, we define relation between production density $S(x,t)$ and we call S_{im} as a synchronization production density. In other word, S_{im} indicates a critical point of phase transition.

We define an order parameter known as a variable that causes phase transition as follows.

Definition 4.3. *Order parameter: production density function $S_i(x,t)$.*

Definition 4.4. *State of each phase*

$$\begin{cases} S_i(x,t) > S_{im} & \textit{Excess inventory} \\ S_i(x,t) = S_{im} & \textit{Syncronized throughput} \\ S_i(x,t) < S_{im} & \textit{Lost opportunity} \end{cases} \tag{9}$$

where S_{im} indicates a production density in the bottleneck, and let it be called as a synchronization production density of output in a whole system. It can be expressed by

$$S_{im} = < \int_0^t f(S_i,\tau)d\tau > \tag{10}$$

where $< \circ >$ indicates a time average.

5. **Ginzburg-Landau Free Energy in Manufacturing Process.** Here, description will be made about what Ginzburg-Landau free energy (hereinafter, referred to as G-L free energy) in manufacturing industry is like.

Definition 5.1. *Free energy: $F(S_i)$ related to production quantity*

$$F(S_i) = \int_0^L \left[f(S_i) + \frac{K_S}{2}(\nabla S_i)^2 \right] dx \tag{11}$$

As previously explained, order parameter $S_i(x,t)$ means a variable related to phase transition. Equation (11) is an equation indicating free energy given by space integration of a function depending on order parameter $S_i(x,t)$, and is called Ginzburg-Landau free energy [9]. Also, $f(S_i)$ indicates a potential function, and ∇S_i represents fluctuation.

Here, a state where Equation (11) becomes minimum in a manufacturing process is found. When $S_i(x,t)$ is changed by $\delta S_i(x,t)$, free energy change $\delta F(S_i)$ is as follows. In this regard, however, space variable $x = [0, L]$ is of one dimension here.

$$\delta F(S_i) = \int_0^L \left[f'(S_i) + D(\nabla S_i(x,t)) \cdot (\nabla \delta S_i(x,t)) \right] dx \tag{12}$$

When partial integration is performed on the second term of Equation (12) taking K_S as a constant, the following equation is obtained.

$$\begin{aligned} &\int_0^L \left[D(\nabla S_i(x,t)) \cdot (\nabla \delta S_i(x,t)) \right] dx \\ &= -\int_0^L \left[D(\nabla^2 S_i(x,t)) \cdot \delta S_i(x,t)) \right] dx \end{aligned} \tag{13}$$

Here, assuming that an order parameter is fixed from the start ($x = 0$) of a manufacturing process to its end ($x = L$), because $\delta S_i(x,t) = 0$, values in the boundary of the partial integration will be also zero. From this, Equation (12) becomes

$$\delta F(S_i) = \int_0^L \left[f'(S_i) - K_S(\nabla^2 S_i(x,t)) \right] \delta S_i(x,t) dx. \tag{14}$$

The condition under which Equation (14) becomes minimum is indicated by

$$f'(S_i) - K_S(\nabla^2 S_i(x,t)) \equiv const. \tag{15}$$

Also, according to the statistical mechanics, assuming that $S_i = 0$ indicates disorderly conditions, it is essentially constituted of second and fourth degree terms of S_i, and a first degree term proportional to an external field. In the present paper, because it is a production field without relation with an external field, a first degree term does not exist. Therefore, let $f(S_i)$ be a function like Equation (16). Figure 6 indicates a case in which two phases (production and non-production) coexist, and, now, it is assumed that G-L free energy has two minimum values S_{im}^1 and S_{im}^2 and they are of an identical energy value. When Equation (16) is viewed as movement of potential, there exists a solution by which $S_{im}(-\infty) = S_{im}^1$ at $x = -\infty$, and $S_{im}(+\infty) = S_{im}^2$ at $x = \infty$. From Equation (15), we obtain

$$f(S_i) = \xi \left\{ -\frac{1}{2} K_C S_i^2 + \frac{K_S}{12} S_i^4 \right\}, \tag{16}$$

where ξ, K_C and K_S are constants.

Now, the relationship between a return and a rate-of-return deviation and profitability production density function,

$$f(S_i(x,t)) = \eta_P h \tag{17}$$

$$\frac{df(S_i(x,t))}{dS_i} = \Delta D \tag{18}$$

where h is a return, and Δ is a rate-of-return deviation.

In addition, according to the statistical mechanics, assuming that we call r as a phase transition factor, we obtain

$$\eta_P r = \left[f''(S_i)/K_S \right]^{1/2}, \tag{19}$$

where η_P means a normalization constant of a potential function. As we have been described before, it can be thought that the number of production densities is proportional to a return. The reason why a return is considered is to consider a boundary width in phase transition in a manner making it correspond to a rate-of-return deviation.

From Equation (19), we obtain

$$r = \frac{1}{\eta_P} \sqrt{\frac{f''(S_i)}{K_S}}. \tag{20}$$

From Equation (16) and Equation (20), we obtain

$$r = \frac{1}{\eta_P} \sqrt{\frac{\xi(-1 + K_S \cdot S_i^2)}{K_S}}, \tag{21}$$

where, $1/r$ indicates a boundary region where two phases of an excessive inventory and an opportunity loss change.

Now, from Equation (18), we obtain

$$\begin{aligned} \Delta D &= \frac{df(S_i(x,t))}{dS_i} \\ &= \sqrt{\xi \left(-K_C S_i + K_S \cdot \frac{1}{3} S_i^3 \right)}. \end{aligned} \tag{22}$$

Potential energy of production density function $H(S_i)$ becomes

$$\begin{aligned} H(S_i) &= \frac{1}{\eta_P} \int_\Omega \xi \left(-K_C S_i + \frac{K_S}{3} S_i^3 \right) dS_i \\ &= \frac{1}{\eta_P} \xi \left(-\frac{1}{2} K_C S_i^2 + \frac{K_S}{12} S_i^4 \right). \end{aligned} \tag{23}$$

At this time, it can be expressed by the following equation according to the degree of order parameter.

$$-S_{im} < S_i(x,t) < +S_{im} \tag{24}$$

6. **Phase Transition in the Flow Production System.** Figure 7 represents a manufacturing process called a flow production system, which is a manufacturing method employed in the production of control equipment. The flow production system, which in this case has six stages, is commercialized by the production of material in steps S1-S6 of the manufacturing process.

The direction of the arrow represents the direction of the production flow. In this system, production materials are supplied from the inlet and the end product will be shipped from the outlet. We make the following two assumptions in this flow production system.

Assumption 2. *The production structure is nonlinear.*

Assumption 3. *The production structure is a closed structure; that is, the production is driven by a cyclic system (flow production system).*

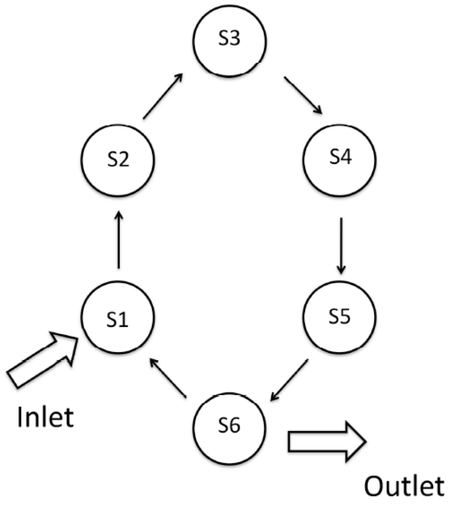

FIGURE 7. Manufacturing flow process

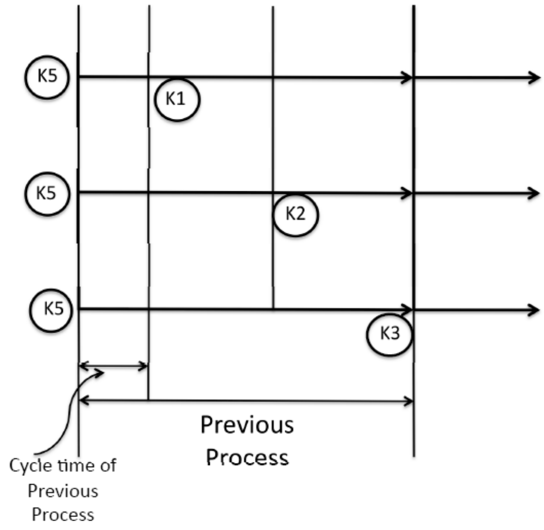

FIGURE 8. Previous process in manufacturing equipment

Assumption 2 indicates that the determination of the production structure is considered a major factor, which includes the generation value of production or the throughput generation structure in a stochastic manufacturing process (hereafter called the manufacturing field). Because such a structure is at least dependent on the demand, it is considered to have a nonlinear structure.

Because the value of such a product depends on the throughput, its production structure is nonlinear. Therefore, Assumption 2 reflects the realistic production structure and is somewhat valid. Assumption 3 is completed in each step and flows from the next step until stage S6 is completed. Assumption 3 is reasonable because new production starts from S1.

Based on the control equipment, the product can be manufactured in one cycle. The production throughput required to maintain 6 pieces of equipment/day is as follows:

$$\frac{(60 \times 8 - 28)}{3} \times \frac{1}{6} \simeq 25 \text{ (min)} \tag{25}$$

where the throughput of the previous process is set as 20 (min). In Equation (25), "28" represents the throughput of the previous process plus the idle time for synchronization. "8" is the number of processes and the total number of all processes is "8" plus the previous process. "60" is given by 20 (min) $\times$ 3 (cycles).

Here, the previous process represents the working until the process itself is entered. To eliminate the idle time after classification of the processes in advance, this previous process was introduced. In Figure 8, for example, it represents the termination of the operation of step K5 during the previous process. By making the corresponding step K5 to be the previous process, there are eight remaining processes. When performing the 3 cycles in Figure 8, the first cycle is $\{\mathrm{K1}, \mathrm{K2}, \mathrm{K3}\}$, the second cycle is $\{\mathrm{K4}, \mathrm{K6}, \mathrm{K7}\}$, and the third cycle is $\{\mathrm{K8}, \mathrm{K9}\}$.

After completion of the third cycle, the workers start manufacturing the next product. That is, the first manufacturing process starts the first cycle. By adopting the previous process cycle, the third cycle is adopted in a parallel process.

At this time, the theoretical throughput ($T_s^{'}$) is as follows.

Here, the previous process is adopted in test-run 5 only.

$$T_s^{'} = 20 \times 6(\text{First cycle}) + 17 \times 6(\text{Second cycle}) \\ + 20 \times 6(\text{Third cycle}) + 20(\text{Previous process}) + 8(\text{Idol-time}) \\ = 364 \text{ (min)} \sim 370 \text{ (min)} \tag{26}$$

One process throughput (20min) in full synchronization is

$$T_s = 3 \times 120 + 40 = 400 \text{ (min)} \tag{27}$$

Therefore, a throughput reduction of about 10% can be achieved. However, the time between processes involves some asynchronous idle time.

As a result, the above test-run is as follows.

- (test-run1): Each throughput in every process (S1-S6) is asynchronous, and its process throughput is asynchronous. Table 1 represents the manufacturing time (min) in each process. Table 2 represents the variance in each process performed by workers. Table 1 represents the target time, and the theoretical throughput is given by $3 \times 199 + 2 \times 15 = 627$ (min).

  In addition, the total working time in stage S3 is 199 (min), which causes a bottleneck. Figure 9 is a graph illustrating the measurement data in Table 1, and it

TABLE 1. Total manufacturing time at each stages for each worker

	WS	S1	S2	S3	S4	S5	S6
K1	15	20	20	25	20	20	20
K2	20	22	21	22	21	19	20
K3	10	20	26	25	22	22	26
K4	20	17	15	19	18	16	18
K5	15	15	20	18	16	15	15
K6	15	15	15	15	15	15	15
K7	15	20	20	30	20	21	20
K8	20	29	33	30	29	32	33
K9	15	14	14	15	14	14	14
Total	145	172	184	199	175	174	181

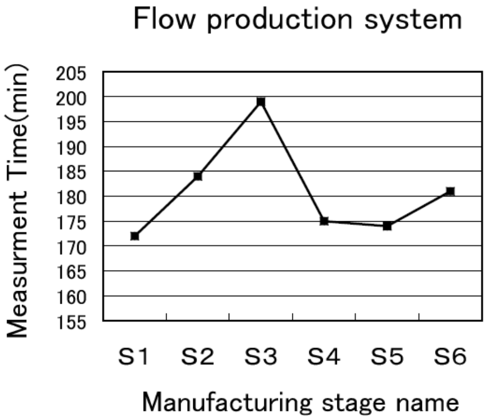

FIGURE 9. Total manufacturing time at each stages for each worker

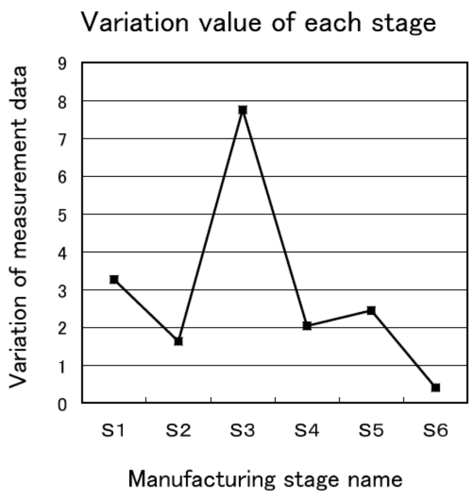

FIGURE 10. Variance data for each worker at each stages

Table 2. Variance of Table 1

K1	1.67	1.67	3.33	1.67	1.67	1.67
K2	2.33	2	2.33	2	1.33	1.67
K3	1.67	3.67	3.33	2.33	2.33	3.67
K4	0.67	0	1.33	1	0.33	1
K5	0	1.67	1	0.33	0	0
K6	0	0	0	0	0	0
K7	1.67	1.67	5	1.67	2	1.67
K8	4.67	6	5	4.67	5.67	6
K9	0.33	0.33	0	0.33	0.33	0.33

Table 3. Total manufacturing time at each stages for each worker

	WS	S1	S2	S3	S4	S5	S6
K1	20	20	24	20	20	20	20
K2	20	20	20	20	20	22	20
K3	20	20	20	20	20	20	20
K4	20	25	25	20	20	20	20
K5	20	20	20	20	20	20	20
K6	20	20	20	20	20	20	20
K7	20	20	20	20	20	20	20
K8	20	27	27	22	23	20	20
K9	20	20	20	20	20	20	20
Total	180	192	196	182	183	182	180

Table 4. Variance of Table 3

K1	0	1.33	0	0	0	0
K2	0	0	0	0	0.67	0
K3	0	0	0	0	0	0
K4	1.67	1.67	0	0	0	0
K5	0	0	0	0	0	0
K6	0	0	0	0	0	0
K7	0	0	0	0	0	0
K8	2.33	2.33	0.67	1	0	0
K9	0	0	0	0	0	0

represents the total working time for each worker (K1-K9). The graph in Figure 10 represents the variance data for each working time in Table 1.

- (test-run2): Set to synchronously process the throughput.

  The target time in Table 3 is 500 (min), and the theoretical throughput (not including the synchronized idle time) is 400 (min). Table 4 represents the variance data of each working process (S1-S6) for each worker (K1-K9).
- (test-run3): The process throughput is performed synchronously with the reclassification of the process. The theoretical throughput (not including the synchronized idle time) is 400 (min) in Table 5.

  From this result, the idle time must be set at 100 (min). Based on the above results, the target theoretical throughput (T_s') is obtained using the shift throughput

TABLE 5. Total manufacturing time at each stages for each worker

	WS	S1	S2	S3	S4	S5	S6
K1	20	18	19	18	20	20	20
K2	20	18	18	18	20	20	20
K3	20	21	21	21	20	20	20
K4	20	13	11	11	20	20	20
K5	20	16	16	17	20	20	20
K6	20	18	18	18	20	20	20
K7	20	14	14	13	20	20	20
K8	20	22	22	20	20	20	20
K9	20	25	25	25	20	20	20
Total	180	165	164	161	180	180	180

TABLE 6. Variance of Table 5

K1	0.67	0.33	0.67	0	0	0
K2	0.67	0.67	0.67	0	0	0
K3	0.33	0.33	0.33	0	0	0
K4	2.33	3	3	0	0	0
K5	1.33	1.33	1	0	0	0
K6	0.67	0.67	0.67	0	0	0
K7	2	2	2.33	0	0	0
K8	0.67	0.67	0	0	0	0
K9	1.67	1.67	1.67	0	0	0

TABLE 7. Total manufacturing time at each stages for each worker, *: Lower than set value

	WS	S1	S2	S3	S4	S5	S6
K1	20	18	19	18	18	18	18
K2	20	18	18	18	18	18	18
K3	20	21	21	21	21	21	21
K4	*16	13	11	11	13	13	13
K5	16	16	16	17	17	16	16
K6	16	18	18	18	18	18	18
K7	20	14	14	13	14	14	13
K8	20	22	22	22	22	22	22
K9	20	20	20	20	20	20	20
Total	168	165	164	163	166	165	164

method. This goal is

$$T_s' = 120 \times 2 + 96 + 24 = 360 \text{ (min)} \tag{28}$$

After T_s' plus the idle time, the throughput (T_s) is

$$T_s = 360 + 100 = 460 \text{ (min)} \tag{29}$$

Table 6 represents the variance data of Table 5.

- (test-run4): Set the shift throughput in some of the processes.

TABLE 8. Variance of Table 7

K1	0.67	0.33	0.67	0.67	0.67	0.67
K2	0.67	0.67	0.67	0.67	0.67	0.67
K3	0.33	0.33	0.33	0.33	0.33	0.33
K4	1	1.67	1.67	1	1	1
K5	0	0	0.33	0.33	0	0
K6	0.67	0.67	0.67	0.67	0.67	0.67
K7	2	2	2.33	2	2	2.33
K8	0.67	0.67	0.67	0.67	0.67	0.67
K9	1.67	1.67	1.67	1.67	1.67	1.67

TABLE 9. Total manufacturing time at each stages for each worker, K5: Previous process, *: Lower than set value, (K4, K6, K7): Second cycle

	WS	S1	S2	S3	S4	S5	S6
K1	20	18	19	18	18	18	18
K2	20	18	18	18	18	18	18
K3	20	21	21	21	21	21	21
K4	*16	13	11	11	13	13	13
K5	16	*	*	*	*	*	*
K6	16	18	18	18	18	18	18
K7	16	14	14	13	14	14	13
K8	20	22	22	22	22	22	22
K9	20	20	20	20	20	20	20
Total	148	144	143	141	144	144	143

TABLE 10. Variance of Table 9, K5: Previous process

K1	0.67	0.33	0.67	0.67	0.67	0.67
K2	0.67	0.67	0.67	0.67	0.67	0.67
K3	0.33	0.33	0.33	0.33	0.33	0.33
K4	1	1.67	1.67	1	1	1
K5	*	*	*	*	*	*
K6	0.67	0.67	0.67	0.67	0.67	0.67
K7	0.67	0.67	1	0.67	0.67	1
K8	0.67	0.67	0.67	0.67	0.67	0.67
K9	0	0	0	0	0	0

The target time is 500 (min) in Table 7, and the theoretical throughput (not including the synchronized idle time) is 400 (min).

From this result, set the idle time at 80 (min). Based on the above result, the theoretical throughput (T_s') obtained using the shift throughput method is

$$T_s' = 400 + 80 = 480 \text{ (min)} \tag{30}$$

Table 8 represents the variance data for Table 7.

- (test-run5): Set the shift throughput in some of the processes.

Table 9 represents the same result of test-run4, and the theoretical throughput (not including the synchronized idle time) is 400 (min). From this result, the idle time must be set at 80 (min). Based on this result, the theoretical throughput ($T_s^{'}$) obtained using the shift throughput method is as follows.

"WS" in the measurement tables represents the standard working time. This is an empirical value obtained from long-term experiments.

$$T_s^{'} = 400 + 80 = 480 \text{ (min)} \tag{31}$$

Table 10 represents the variance data of Table 9.

The results are as follows.

Here, the trend coefficient, which is the actual number of pieces of equipment/the target number of equipment, represents a factor that indicates the degree of the number of pieces of manufacturing equipment.

Test-run1: 4.4 (pieces of equipments)/6 (pieces of equipments) = 0.73, Test2-3 = 5.5 (pieces of equipments)/6 (pieces of equipments) = 0.92, Test4-5 = 5.7 (pieces of equipments)/6 (pieces of equipments) = 0.95.

Variance data represent the average value of each test-run.

The read-time with test-run1 in Table 11 decreases from 627 (min) to 500 (min), and the data shows that the phase transition occurs accurately.

7. **Numerical Example of a Phase Transition Coefficient.** As explained above, we found that the trend coefficient in Table 11 varies greatly between test-run1 and test-run2. In other words, it is understood that the average number of pieces of manufacturing equipment increases from 4.4 (pieces of equipment) to 5.5 (pieces of equipment). This trend coefficient represents a factor corresponding to η_P in Table 12.

K_S in Table 12 represents a phase transition coefficient (r), and both η_P and η_S represent a normalized constant.

Figure 11 illustrates the change in the phase transition factor when ΔD_n varies from -0.3 to $+0.3$, and $1/r$ indicates a boundary area in which phases are changed. In the horizontal axes of Figures 11-13, the area indicating $1/r$ is an area near $-0.133 < 1/r < +0.1$. Figure 12 indicates a change in the potential value, and Figure 13 indicates a change in the rate-of-return deviation. In Figures 12 and 13, as ΔD_n becomes larger, $1/r$ also becomes larger, and as a result, the boundary region becomes wider. That is, a lower percentage is required to cause the phase transition.

TABLE 11. Test-running results

	Trend coefficient	Variance	Read-Time
Test1	0.73	0.29	627
Test2	0.92	0.06	500
Test3	0.92	0.03	500
Test4	0.95	0.03	480
Test5	0.95	0.03	480

TABLE 12. Set parameter values

	Type-1	Type-2	Type-3	Type-4	Type-5	Type-6	Type-7	Type-8	Type-9	Type-10
$\backslash\ K_S$	0.05	0.06	0.07	0.08	0.09	0.1	0.2	0.3	0.4	0.5
η_P	100	100	100	100	100	100	200	500	600	1000
η_S	10	10	10	10	10	100	100	250	300	500
η_S/η_P	0.1	0.1	0.1	0.1	0.1	1.0	0.5	0.5	0.5	0.5

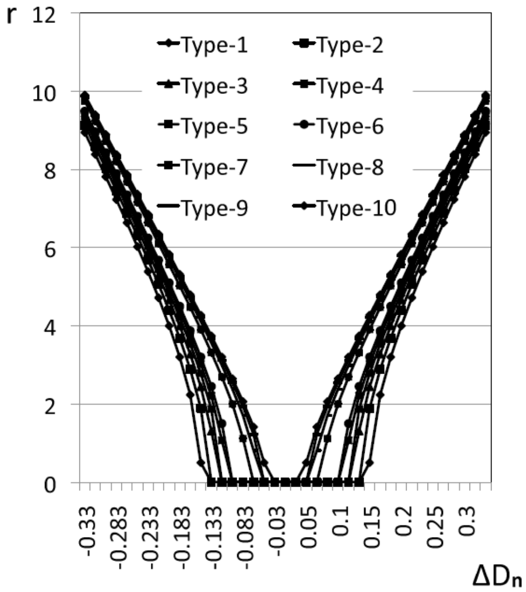

FIGURE 11. The phase transition factor values r corresponding to the normalization value of rate-of-return deviation ΔD_n

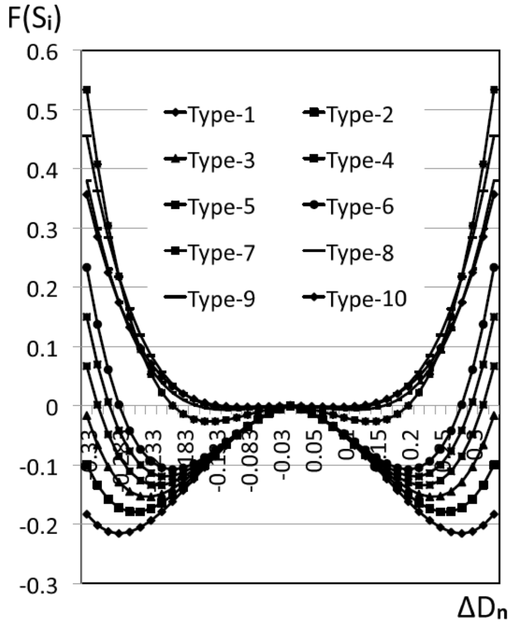

FIGURE 12. The potential values $F(S_i)$ corresponding to the normalization value of rate-of-return deviation ΔD_n

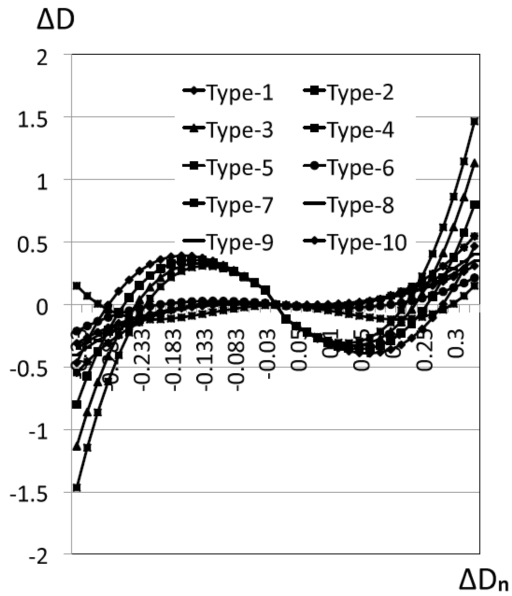

FIGURE 13. The rate-of-return deviation values ΔD corresponding to the normalization value of rate-of-return deviation ΔD_n

The normalization constant was changed such that the normalization value of the rate-of-return deviation ΔD_n lies within the range -0.3 to $+0.3$. This is because by doing so, the relationship between the phase transition and potential becomes clear.

Accordingly, we think that it is important to know $|\Delta D_n|$, which is a critical point of the phase transition, and invest heavily into manufacturing operations in order to ensure the continued viability of the business.

8. **Conclusion.** In this study, we performed a statistical mechanical analysis of manufacturing processes in the manufacturing industry and analyzed a mechanism for the phase transition on the premise that there exists a phase transition phenomenon in manufacturing processes. We simulated several manufacturing processes so that when actual manufacturing processes are seen from the perspective of the rate-of-return deviation, there remains a range of values that represent critical points in the phase transition, which in this case, falls within $-0.3 < \Delta D_n < 0.3$. In this numerical value calculation example, we found that the boundary width of the phase transition is approximately the width of the region $-0.13 < 1/r < 0.1$. Although the rate-of-return differs based on the system, the rate-of-return is improved by setting the "Edge of Chaos", that is, the width of the rate-of-return deviation, and maintaining that value. This is particularly important to ensure continuity of business operations.

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