Construction of a cluster model of the production process using division vectors

Kenji Shirai[1] Yoshinori Amano[2]

Abstract

Previously, we have proposed a lot-casting process, namely, the asymmetric simple exclusion process (ASEP), to improve throughput. The main cause of production retention between processes is the fact that different processes operate at different speeds, i.e., the problem of working ability. In the present article, we describe the theoretical background of our previously proposed ASEP model. Numerous human workers carry out production according to the rules of the production flow system. During this flow, no worker progresses to the next process until the front process ends. Therefore, data can be updated one by one from the front step. Furthermore, it is assumed that either the system does not know the number of clusters in advance or the number of clusters differs with time.

Keyword: cluster model, division vector, asymmetric simple exclusion process, Dirichlet distribution

1 Introduction

In our previous report, we calculated the expected loss value from a lead-time function and verified a loss function with actual data[1]. Moreover, through theoretical analysis, we clarified that for improved productivity, minimizing both volatility and fixed costs is important. Comparing synchronous and asynchronous methods revealed a reduction of approximately 10% in the results when using synchronous throughput. A simple exclusion process is the one-dimensional (1-D) non-equilibrium statistical mechanics model, known as the asymmetric simple exclusion process (ASEP). ASEP is used in production lines to improve production efficiency. As an application method, ASEP is used to optimize the production lot[2, 3].

We have also used ASEP to improve the efficiency of the production process. When applied as a model of a lot production system, ASEP is fundamentally a nonlinear system represented by Burgers' equation. This indicates that the process transition probability plays an important role. Based on the experience of conducting three test runs of the production flow process (PFP), we clarify the characteristics of each process and select the appropriate value[6, 7]. We appropriately arrange the workers in each process. A highly linear system is obtained from the aforementioned procedure that approaches a stationary system.

Dr. Nishinari proposed the ASEP model, the theoretical predictions of which are consistent with simulation results, thereby validating the model. We apply the ASEP model to an actual production process and also present the comparison of the actual data with PFP. Comparing the ASEP and PFP production efficiencies, ASEP was able to double the throughput and reduce production costs by 20%[3]. In the present study, we propose a mathematical model using a division-vector concept for ASEP. The production process comprises numerous workers and processes, which we refer to as the production space. Consider the production space as a division vector, which is the equipment to be produced. The elements of the division vector correspond to the lead time, and each element is probabilistic. Therefore, we can obtain the distribution of occurrence probability using the division vector without conducting a finite number of tests. The assumptions regarding the division-vector model are as follows:

- The process transition is performed independently for each step.

- There is no correlation concerning the transition of each step.

- The production process primarily depends on the probability in the human work.

---

[1]Department of Business Administration Faculty of Business and Informatics
[2]Kyohnan Elecs co.,LTD

2　Production framework in equipment manufacturer

We refer to the production system in manufacturing equipment industry studied in this paper. This is not a special system but "Make-to-order system with version control". Make-to-order system is a system which allows necessary manufacturing after taking orders from clients, resulting in "volatility" according to its delivery date and lead time. In addition, "volatility" occur in lead time depending on the contents of make-to-order products (production equipment).

However, effective utilization of the production forecast information on the orders may suppress certain amount of "variation", but the complete suppression of variation will be difficult. In other words, "volatility" in monthly cash flow occurs and of course influences a rate of return in these companies. Production management systems, suitable for the separate make-to-order system which is managed by numbers assigned to each product upon order, is called as "product number management system" and is widely used. All productions are controlled with numbered products and instructions are given for each numbered products.

Thus, ordering design, logistics and suppliers are conducted for each manufacturer's serial numbers in most cases except for semifinished products (unit incorporated into the final product) and strategic stocks. Therefore, careful management of the lead time or production date may not suppress "volatility" in manufacturing (production). The company in this study is the "supplier" in Figure 1 and "factory" here. Companies are under the assumption that there are N (numbers of) suppliers; however, this study deals with one company because no data is published for the rest of the companies $(N-1)$.

2.1　ASEP production processes

The ASEP is a non-equilibrium statistical mechanics model that is referred to as an exclusive process. It is a simple model in which numerous particles diffuse under the volume-exclusion interaction on a one-dimensional lattice. Figure 2 shows a one-lot-flowing model in ASEP. The terms α and β denote the input and output rates, respectively, and p denotes the transition probability from one stage to another[3].

In Figure 3, K1 to K5 are worker numbers, and there is no determined work order. The "consumption time" represents the in-process consumption time when continuous production is performed. The total lead time is the processing time of one lot, namely, the total time taken to go through steps 1–5 plus the time for the remaining 11 to pass through step 5. The theoretical value is derived as follows:

$$(5 \ \times \ 25) \ + \ (25 \ \times \ 11) \ = \ 400 \tag{2.1}$$

With respect to the lead time, we obtain the following equation according to Dr. Nishinari[2]. Please refer to the reference for detailed information[3].

$$T = \left[\left(\frac{1}{\frac{1}{25}}\right) + \left(\frac{1}{\frac{1}{25}}\right)\right] + \left(\frac{11}{\frac{1}{25}}\right) + \left(\frac{5}{\frac{1}{25}}\right) \tag{2.2}$$

where, $\frac{1}{25}$ denotes a transition rate. The first and second term denote an input/output time.

3　System features

In Figure 4, we select N from the population, which has elements identified as K types $(a_1, \cdots, a_K)$. Furthermore, it is classified into K classes $(m_1, \cdots, m_K)$. The appearance ratio of the K elements is stochastic; in other words, the appearance ratio varies with the test. The production space contains the vector $\boldsymbol{a} = a_1(n_1), \cdots, a_K(n_K)$. When the data updated from the population belong to the original class, vector $\boldsymbol{a}$ remains unchanged. If the original and new data are replaced, the replacement destination vector is set to $+1$, and the vector before replacement is set to -1. Consequently, the total number of pieces does not change. However, in Figure 5, if the data updated from the population belong to the original
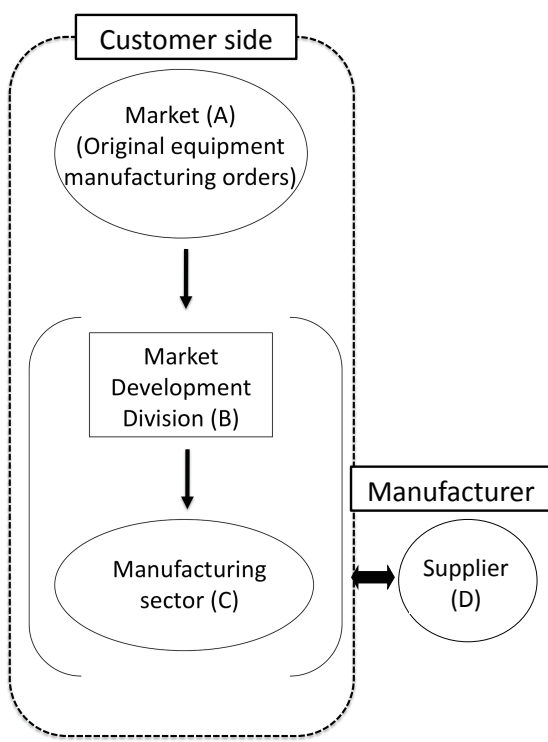
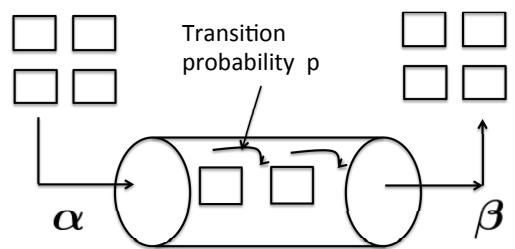Figure 1: Business association chart of company of re-search target
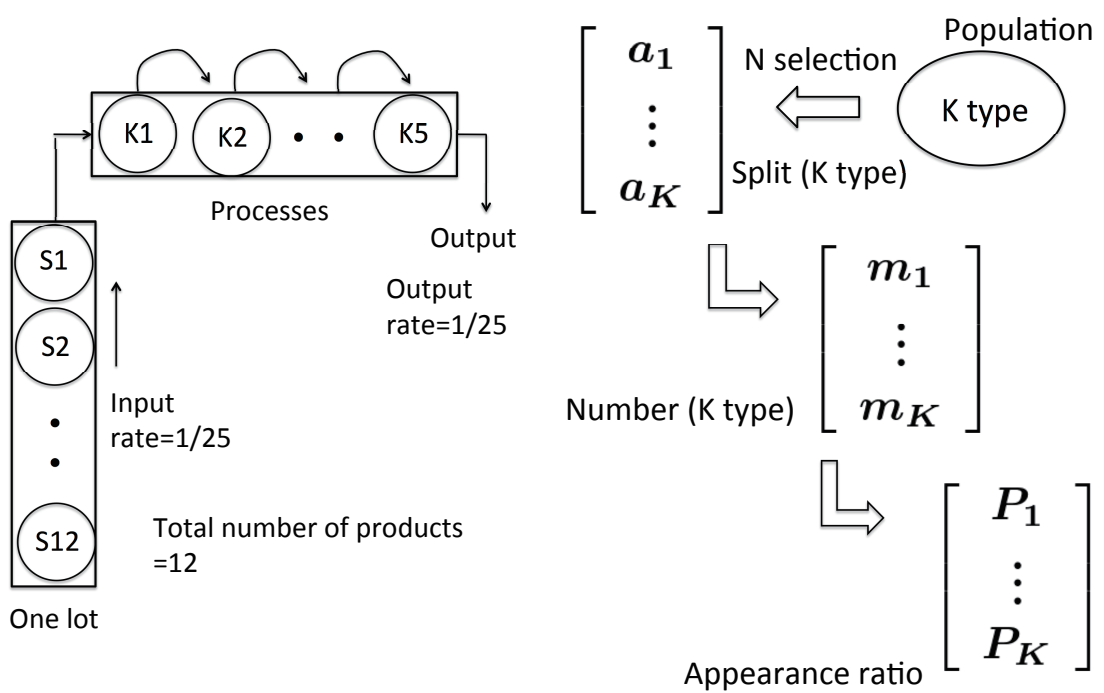

Figure 2: Asep model

Processes

K1 K2 • • K5

Output

Output
rate=1/25

S1

S2

Input
rate=1/25

S12

Total number of products
=12

One lot

Figure 3: ASEP production model

Population

$$\begin{bmatrix} a_1 \\ \vdots \\ a_K \end{bmatrix}$$

N selection

K type

Split (K type)

$$\begin{bmatrix} m_1 \\ \vdots \\ m_K \end{bmatrix}$$

Number (K type)

$$\begin{bmatrix} P_1 \\ \vdots \\ P_K \end{bmatrix}$$

Appearance ratio

Figure 4: Choose different production factors of K type for each test stochastically
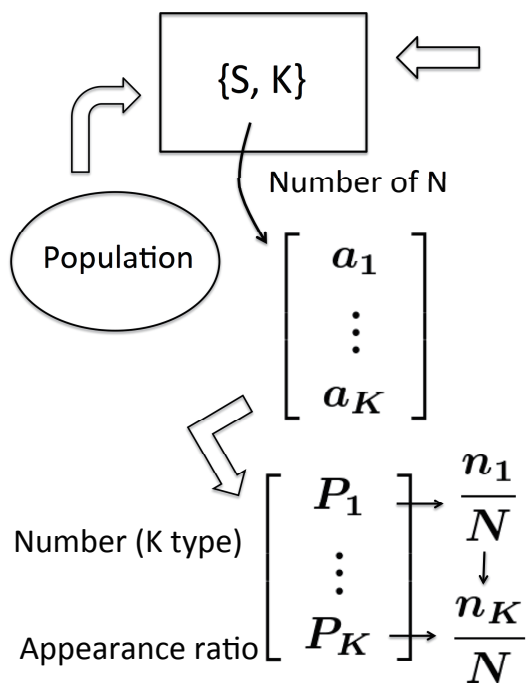
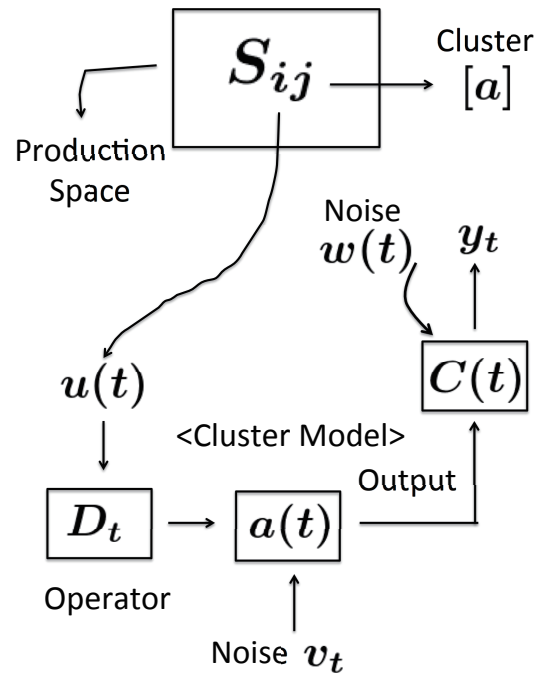Figure 5: Transition from population to appearance probability

Figure 6: System model concept using cluster space

class, vector $\boldsymbol{a}$ should be left as it is; if not, the vector of the updated destination should be set to $+1$, and the vector before updating should be set to -1. The total number does not change.

$$\sum_{i=1}^{K} a_i = N \tag{3.1}$$

$$\sum_{i=1}^{K} n_i = N \tag{3.2}$$

Then, $p_i(a_i) = \frac{n_i}{N}, i = 1, 2, \cdots, K$, where p_i is the proportion of a_i of cost in the whole and follows the Dirkle distribution. Here, the production space is denoted by $\{S, K\}$, where S and K represent a product and a process, respectively. Each element is a_{ij}, and the total number is $[i] \times [j] = N$. However, each element is assumed to be classified as K type. At this time, n_i becomes as follows.

Assumption 3.1

$$n_1 + n_2 + \cdots + n_K = N \tag{3.3}$$

Here, the vector expressing the number of elements of this type is as follows.

$$\boldsymbol{n} = (n_1, n_2, \cdots, n_K) \tag{3.4}$$

Let us now assume that when $K = 3$, we consider a unit vector with the j-th vector e_j, and only the j-th element is one. For the above example, e_j is a k-dimensional vector whose j-th component is one and the other is zero.

$$\boldsymbol{e}_1 = (1, 0, 0), \quad \boldsymbol{e}_2 = (0, 1, 0) \tag{3.5}$$

Here, the rate at which the $j-$ type element is excluded (exited) from the production space is assumed as follows.

Assumption 3.2

$$w(\boldsymbol{m}, \boldsymbol{n} - e_j) = c_j \frac{n_j}{\boldsymbol{n}}, \quad j = 1, 2, \cdots, K \tag{3.6}$$

where,

$$\boldsymbol{\alpha} = \alpha_1 + \alpha_2 + \cdots + \alpha_K, \quad \alpha_j > 0 \tag{3.7}$$

where, α is the type to be replaced.

Furthermore, when an element of type i changes to type j, if its rate is such that type i is eliminated and type j equals the rate to be replaced, the rate to be replaced is as follows:

$$w(\boldsymbol{n}, \boldsymbol{n} - e_i + e_j) = w(\boldsymbol{n}, \boldsymbol{n} - e_i)w(\boldsymbol{n} - e_i, \boldsymbol{n} - e_i + e_j)$$
$$= c_i \frac{n_i}{\boldsymbol{n}} d_j \frac{n_j + \alpha_j}{\boldsymbol{n} + \boldsymbol{\alpha} - 1} \tag{3.8}$$

The element α_j in $\boldsymbol{\alpha}$ is replaced in Equation (3.8). The opposite case can be obtained in the same way. Assuming that $\boldsymbol{n}' = \boldsymbol{n} + e_i - e_j$, $\boldsymbol{n} = \boldsymbol{n}' - e_i + e_j$. Therefore,

$$w(\boldsymbol{n}, \boldsymbol{n} - e_i + e_j) = w(\boldsymbol{n}, \boldsymbol{n} - e_i)w(\boldsymbol{n} - e_i, \boldsymbol{n} - e_i + e_j)$$
$$= c_i \frac{n_i}{\boldsymbol{n}} d_j \frac{n_j + \alpha_j}{\boldsymbol{n} + \boldsymbol{\alpha} - 1} \tag{3.9}$$

The balance of replacement is as follows.

$$\pi(\boldsymbol{n}) = c_i \frac{n_j}{\boldsymbol{n}} d_j \frac{n_i + \alpha_i}{\boldsymbol{n} + \boldsymbol{\alpha} - 1}$$

$$= \pi(\boldsymbol{n}') c_i \frac{n_j - 1}{\boldsymbol{n}} d_j \frac{n_j + \alpha_j - 1}{\boldsymbol{n} + \boldsymbol{\alpha} - 1} \tag{3.10}$$

We assume that a stationary distribution is as follows.

Assumption 3.3

$$\pi(\boldsymbol{n}) = \pi_1(n_1)\pi_2(n_2)\cdots\pi_K(n_K) \tag{3.11}$$

We obtain as follows.

$$\pi_i(n_i)\pi_j(n_j)c_j \frac{n_j}{\boldsymbol{n}} d_i \frac{n_i + \alpha_i}{\boldsymbol{n} + \boldsymbol{\alpha} - 1}$$

$$= \pi_i(n_i + 1)\pi_j(n_j - 1)c_i \frac{n_j + 1}{\boldsymbol{n}} d_i \frac{n_j + \alpha_j - 1}{\boldsymbol{n} + \boldsymbol{\alpha} - 1} \tag{3.12}$$

Therefore, according to the recurrence formula of n_i and n_j, we obtain as follows.

$$\frac{\pi_i(n_i + 1)}{\pi_i(n_i)} = B_i g_i \frac{n_i + \alpha_i}{n_i + 1}$$

$$\frac{\pi_i(n_j)}{\pi_j(n_j - 1)} = B_j g_j \frac{n_j - 1\alpha_j}{n_j} \tag{3.13}$$

where, B_i and B_j are an undetermined multiplier so that a probability distribution of $\pi_i(n_i)$ and $\pi_j(n_j)$ exist.

$$g_i = \frac{c_i}{d_i} \tag{3.14}$$

Then, $\pi_i(n_i)$ is set as follows.

$$\sum_{n_i} \pi_i(n_i) = 1 \tag{3.15}$$

The probability distribution of the number n_i of elements of type i is as follows.

$$\pi_i(n_i) = (1 - g_i)^{\alpha_i} \frac{\alpha_i^{[n_i]}}{n_i!} g_i^{n_i} \tag{3.16}$$

where, $\alpha_i^{[n_i]}$ is derived as follows.

$$\alpha_i^{[n_i]} = \alpha_i \cdots (\alpha_i + 1) \cdots (\alpha_i + n_i - 1) \tag{3.17}$$

Therefore, the probability distribution of the number of elements of each type is as follows.

$$\pi_i(n_i) = (1 - g_i)^{\alpha_i}(-g_i)^{n_i}\left(\frac{-\alpha_i}{n_i}\right), \quad i = 1, 2, \cdots, K \tag{3.18}$$

Equation (3.18) represents a negative binomial distribution. Equation (3.18) presents the average of elements of type i as follows.

$$E[n_i] = \alpha_i \frac{1 - \alpha_i}{g_i} \tag{3.19}$$

Therefore, the average of the K type is as follows.

$$E[N] = \sum_{i=1}^{K} \alpha_i \frac{1 - g_i}{g_i} \tag{3.20}$$

For Equation (3.20) to take a positive value, $g_i < 1$. In case of $g_i = 1(c_i = d_i)$, the following can be derived.

$$\pi(\boldsymbol{n}) = \frac{n!}{\alpha^{[n]}} \prod_{i=1}^{K} \frac{\alpha_i^{[n_i]}}{n_i!} \tag{3.21}$$

Equation (3.21) is s called a polyar distribution function. Theoretical development so far aimed at reducing costs or reducing losses. In other words, we aim to improve the cost of production system through elimination and introduction of elements categorized by such type.

When n is large, it becomes a Dirichlet distribution through Sterling's formula:

$$\pi(\boldsymbol{n}) = \frac{n!}{n_1! \cdots n_K!} \frac{\prod_{i=1}^{n} \alpha_i^{[n_i]}}{\alpha^{[n]}} \tag{3.22}$$

We define the replacement probability as follows.

Definition 3.1 *Replacement probability*

$$P(\boldsymbol{n} + e_j | \boldsymbol{n}) = f_j(n_j, n) = \frac{\alpha_j + n_j}{\alpha + n} \tag{3.23}$$

We define the exclusion probability as follows.

Definition 3.2 *Exclusion probability*

$$P(\boldsymbol{n} - e_K | \boldsymbol{n}) = \frac{n_K}{n} \tag{3.24}$$

The following is assumed.

$$P(\boldsymbol{n} + e_i - e_j) = P(\boldsymbol{n} - e_j + e_i) \tag{3.25}$$

From Equation (3.25), we obtain as follows.

$$\begin{aligned} P(\boldsymbol{n} + e_i - e_j | \boldsymbol{n}) &= P(\boldsymbol{n} + e_i | \boldsymbol{n}) P(\boldsymbol{n} + e_i - e_j | \boldsymbol{n} + e_i) \\ &= P(\boldsymbol{n} - e_j | \boldsymbol{n}) P(\boldsymbol{n} + e_i - e_j | \boldsymbol{n} + e_i) \end{aligned} \tag{3.26}$$

Now, we assume that $\alpha_1 = \alpha_2 = \cdots = \alpha_K$. We can modify Equation (3.22) as follows.

$$P(\boldsymbol{n}) = \frac{n!}{(k\beta)^{[n]}} \prod_{i=1}^{K} \frac{\beta^{[n]}}{n_j} \tag{3.27}$$

where, $\sum_{i=1}^{K} = k\beta$.

Generally, let it be $(P_1, P_2, \cdots, P_K)$. If there are n_j elements of type $[j]$ among N samples, if the mixed states are $\boldsymbol{n} = (n_1, n_2, \cdots, n_K)$, the probability of vector $\boldsymbol{n}$ is as follows.

$$P(\boldsymbol{n}) = \frac{n!}{n_1! \cdots n_K!} P_1^{n_K} P_2^{n_K} \cdots P_K^{n_K} \tag{3.28}$$

where, we assume that $P_i(i = 1, 2, \cdots, K)$ is distributed on the K-dimensional simplex (Δ).

Assumption 3.4

$$\Delta = \{P_i \geq 0, \quad i = 1, 2, \cdots, K, \quad \sum_{i=1}^{K} P_i = 1\} \tag{3.29}$$

In the production system developed in the production space $\{S, K\}$, type-specific elements that are lead times appear stochastically based on the ability distribution of each worker. Therefore, we can obtain the above distribution without conducting a finite number of tests. We also consider using the general Dirichlet distribution theory below. Now, let θ_j be the probability that the ratio p_j, $j = 1, 2, \cdots, m$ of type elements appears. At this instance, when the Dirichlet distribution is used, the appearance probability distribution becomes as follows:

$$P(\theta) = \frac{\Gamma\left(\sum_j p_j\right)}{\prod_j \Gamma(p_j)} \prod_j \theta_j^{p_j - 1} \tag{3.30}$$

With respect to $\prod_j \theta^{\alpha_j - 1}$, we obtain as follows by integrating mathematically the simplex method of above description.

$$\int_{\theta | \sum_j \theta_j = 1, \theta_j \geq 0} \prod_j \theta_j^{p_j - 1} d\theta = \int_0^1 \theta_1^{p_1 - 1} d\theta_1 \cdot \int_0^{1 - \theta_1} \theta_2^{p_2 - 1} d\theta_2 \cdots$$
$$\int_0^{1 - \theta_1 - \cdots \theta_{m-2}} \theta_{m-1}^{p_{m-1} - 1} d\theta_{m-1} \int_0^{1 - \theta_1 - \cdots \theta_{m-1}} \theta_m^{p_m - 1} d\theta_m \tag{3.31}$$

The terms of normal distribution are obtained by sequentially executing the above calculation. We will treat the probability distribution $P_K = 1 - p_1 \cdots p_{K-1}$ of $K - 1$ dimension defined by the simplex method as Δ. At this instance, when the probability density function uses the parameter α, it becomes as follows:

$$f(p) \cong \prod_{i=1}^{K} p_i^{\alpha_i - 1} \tag{3.32}$$

where,

$$\alpha_{ij} = \sum_{l=1}^{j} \alpha_l, \quad A = \alpha_{1;K} \tag{3.33}$$

The average, variance and Covariance of Dirichlet distribution are derived as follows.

$$E(P) = \frac{\alpha}{A}$$
$$V(p_i) = \frac{\alpha_i(A - \alpha_i)}{A^2(A+1)}$$
$$Cov.(p_i, p_j) = \frac{-\alpha_i \alpha_j}{A^2(A+1)}, \quad i \neq j \tag{3.34}$$

We can obtain as follows by normalization $f(P)$.

$$f(P) = \frac{1}{B(\alpha)} \prod_{i=1}^{K} p_i^{\alpha - 1} \tag{3.35}$$

where $B(\alpha)$ denotes as follows.

$$B(\alpha) = \frac{\prod_{i=1}^{K} \Gamma(\alpha)}{\Gamma(A)} \tag{3.36}$$

Equation (3.34) is derived by utilizing Equations (3.35) and (3.36).

The system features are as follows.

- We classified products to be produced by type.

- We classified workers (groups) for each classified product by ability.

- Synchronization process was considered when constructing the production system.

Here, the products are classified into N types of products. Thus, there exist N types of product having K different types of part. For example, $a_k = N$, where K and N represent part type and product type, respectively. Such products are called division vectors. $b_{i,j}$ means that the same type of part $b_{i,j}$ exists in the product type a_i; i.e., a_i is the type of product and $b_{i,j}$ is the type of component used in the product b-type.

4 System model with division vector as state vector

In Figure 6, the production factor S_{ij} is classified into a type-specific vector $\boldsymbol{a}$. We update the production factors according to the updated rules. The system observes y_t to estimate the system state. The state vector $\boldsymbol{a}$ and noise vector $\boldsymbol{v}$ are defined as follows.

Definition 4.1 *State vector* $\boldsymbol{a}$

$$\boldsymbol{a} = \begin{bmatrix} a_1(t) \\ \vdots \\ a_i(t) \\ \vdots \\ a_K(t) \end{bmatrix} \tag{4.1}$$

Definition 4.2 *Noise vector* $\boldsymbol{v}$

$$\boldsymbol{v} = \begin{bmatrix} v_1(t) \\ \vdots \\ v_i(t) \\ \vdots \\ v_K(t) \end{bmatrix} \tag{4.2}$$

where, K is a number of state.

$$\boldsymbol{a} \equiv (a(t-1), v(t)), \quad \boldsymbol{v} \equiv P^v(v_t|\theta_t^v) \tag{4.3}$$

The noise $\boldsymbol{v}(t)$ is added at the time of generating the state vector. K is a number of state.

Definition 4.3 *Different unknown parameters for each time point*

$$\boldsymbol{\theta}_t^v \equiv (\theta_1^v, \cdots, \theta_i^v, \cdots, \theta_K^v) \tag{4.4}$$

$\boldsymbol{\theta}_t^v$ has noise v_t and changes with time. The mixed distribution $P^v(v_t|\theta_t^v)$ is that the probability density distribution is modeled by weighted linear combination of n probability density distributions.

Definition 4.4 *Dirichlet distribution* G^v

$$G^v \sim DP(G_0^v, r^v) \tag{4.5}$$

where, D is Dirichlet process. G_0^v and r^v are a base distribution and a scaling factor respectively.

Assumption 4.1 *Determination of unknown parameter* θ_i^y

$$\theta_i^y | G^v \sim G^v, \quad i = 1, 2, \cdots \quad t = 1, 2, \cdots \tag{4.6}$$

$v_i(t)$ is obtained as follows.

$$v_i(t) | \theta_i^y \sim P^v(v_i(t) | \theta_i^y(t)) \tag{4.7}$$

In order to observe the system state, the system is constructed as follows.

Definition 4.5 *Observation vector* $\mathbf{w}$

$$\mathbf{w} = \begin{bmatrix} w_1(t) \\ \vdots \\ w_i(t) \\ \vdots \\ w_K(t) \end{bmatrix} \tag{4.8}$$

$\mathbf{w}$ is defined as follows.

Definition 4.6 *Observation system* $y(t)$

$$y(t) = b(a(t), w(t)), \quad w(t) \equiv P^w(\omega_t | \theta_t^w) \tag{4.9}$$

where, ω_t and θ_t^w are a noise and parameter respectively.

$$G^w \sim DP(G_0^w, r^w), \quad \theta_i^w(t) | G^w \sim G^w \tag{4.10}$$

where, G^w is Dirichlet distribution. $\theta_i^w(t)$ is determined by G^w. Both of G_0^v and G_0^w are a base distribution. Both of r^v and r^w are a scaling factor. G_0^v, G_0^w, r^v and r^w are known.

Definition 4.7 *Hyper parameter* $\varphi(G_0^v, G_0^w, r^v, r^w)$

The system model is derived as follows.

$$a_t = Aa_{t-1} + B_t v_t + G_t v_t$$
$$y_t = C_t a + w_t, \quad w_t \sim N(\mu_t^w, \Sigma_t^w) \tag{4.11}$$

where, $N(\mu_t^w, \Sigma_t^w)$ is a normal distribution with both of average μ_t^w and volatility Σ_t^w.

We describe for confirmation as follows.

$$G^v \sim DP(G_0^v, r^v), \quad \theta_t^y | G^v \sim G^v, \quad, v_t | \theta_t^y \sim N(v_t; \mu_t^v, \Sigma_t^v), \quad t = 1, 2, \cdots \tag{4.12}$$

$$G^w \sim DP(G_0^w, r^w), \quad, \theta_t^w | G^w \sim G^w, \quad w_{it} | \theta_t^w \sim N(w_t; \mu_t^w, \Sigma_t^w), \quad t = 1, 2, \cdots, \tag{4.13}$$

where, both of μ_t^v and μ_t^w are averages. Both of Σ_t^v and Σ_t^w are volatilities. G_0^v and G_0^w follow the inverse Wishart distribution of $(\mu_0^v, \Sigma_0^v, r_0^v)$ and $(\mu_0^w, \Sigma_0^w, r_0^w)$ respectively. μ_0^v and μ_0^w are an initial average of system and initial average of observation system respectively. Σ_0^v and Σ_0^w are an initial volatility of system and initial volatility of observation system respectively.

Definition 4.8 *Hyper parameters* $\varphi^v(\mu_0^v, \Sigma_0^v, r_0^v)$ *and* $\varphi^w(\mu_0^w, \Sigma_0^w, r_0^w)$

Because the system model assumes normality and linearity, when $y_1(t)$ and $\theta_1(t)$ are given, the posterior distribution of $P(a_0(t), \theta_1(t)|y_1(t), \varphi)$ follows a normal distribution, and its mean and covariance matrix can be estimated using a Kalman filter. φ is also a hyper parameter.

Now, the proposed system model from conventional is as follows:

$$\frac{\partial S(t,x)}{\partial t} = \mathscr{L}[S(t,x)] + B(t,x)u(t,x), \quad t \in \Theta, \quad x \in \Omega \tag{4.14}$$

According to Equation (4.14), the cluster model is derived as follows.

$$\frac{d\boldsymbol{a}(t)}{dt} = \boldsymbol{A}(t)\boldsymbol{a}(t) + \boldsymbol{B}(t)\boldsymbol{u}(t) + \boldsymbol{G}\boldsymbol{v}(t) \tag{4.15}$$

where,

$$v(t) \equiv P^v(v(t)|\theta^v(t)), \quad G^v \sim DP(G_0^v, r^v) \tag{4.16}$$

The parameter $\theta^v(t)$ can be denoted as follows.

$$\theta^v(t)|G^v \sim G^v, \quad t \in \Theta', \quad \Theta' \subset \Theta \tag{4.17}$$

where, $v(t) \sim N(\mu_t^v, \Sigma_t^v)$

The output model is described as follows.

$$y(t) = \boldsymbol{C}(t)\boldsymbol{a}(t) + \boldsymbol{w}(t) \tag{4.18}$$

where,

$$w(t) \equiv P^w(w(t)|\theta^w(t)), \quad G^w \sim DP(G_0^w, r^w) \tag{4.19}$$

According to Equation (4.25), the parameter $\theta^w(t)$ is represented as follows.

$$\theta^w(t)|G^w \sim G^w, \quad t \in \Theta' \tag{4.20}$$

where, $w(t) \sim N(\mu_t^w, \Sigma_t^w)$.

Therefore, we obtain the system model as follows.

$$\frac{d\boldsymbol{a}(t)}{dt} = \boldsymbol{A}(t)\boldsymbol{a}(t) + \boldsymbol{B}(t)\boldsymbol{u}(t) + \boldsymbol{G}(t)\boldsymbol{v}(t)$$
$$y(t) = \boldsymbol{C}(t)\boldsymbol{a}(t) + \boldsymbol{w}(t) \tag{4.21}$$

4.1 Example of the cluster model analysis procedure

The assumptions regarding the cluster model are as follows:

- The process transition is performed independently for each step.

- There is no correlation concerning the transition of each step.

- The production process primarily depends on the probability in the human work.

The model analysis procedure is as follows.

- (Standard working time) - (Classified into clusters according to a certain process work time of one worker), that is $\delta C < k(k = 0, 1, 2, 3, 4, 5)$. However, it follows the exchange rule.

- Change element P_i of cluster a_i.

After that, follow the procedure as follows.

let a_i be the i−th element of the division vector $\boldsymbol{a}$, and let a_i be in accordance with the Poisson distribution $P(a_i(t)|\lambda(t))$. Let's set $y_i(t) \equiv P(a_i(t)|\lambda(t))$ again.

$$y_i(t) = a_i(t-1) + v_i(t), \quad v_i(t) \sim N(0, \tau_i^2(t)) \tag{4.22}$$

where, $\tau_i^2(t) \sim IG(s_j, v_j)$. IG is an inverse gamma distribution and v_j is a Mixed Dirichlet process. Let $v_i(t) \sim G^v$. r, initial value $y_i(0)$, $a_i(0)$ and $\tau_i(0)$ are known, which are given by the initial test. $t \in [\tau, \tau + t]$

Now, let $\{\tilde{a}_i\}$ represent the element of a_i occurring in t cycle. This occurrence is subject to the Poisson process.

$$P_r[(\tilde{a}_i^{\tau+t} - \tilde{a}_i^\tau) = k_i] = \frac{e^{-\lambda_i} \cdot \lambda_i^{k_i}}{k_i!} \tag{4.23}$$

Therefore,

$$E_0^P[\tilde{a}_i^{\tau_c}] = \lambda_i, \quad t = \tau_c \tag{4.24}$$

Definition 4.9 *Division vector with cost* $P[\tilde{a}_i^{\tau_c}]$

$$P[\tilde{a}_i^{\tau_c}] = (\beta R_i) E_0^P[\tilde{a}_i^{\tau_c}] \tag{4.25}$$

where, $\beta(0 < \beta < 1)$ is a parameter and R represents the cost per element.

Let the cost rate be r_p.

$$P[\tilde{a}_i^{\tau_c}] = e^{-r_p \tau}(\beta R_i) E_0[\tilde{a}_i^{\tau_c}] \tag{4.26}$$

Let the random variable of $E_0[\tilde{a}_i^{\tau_c}]$ be $x_i(t)$, the state equation representing cost is as follows.

$$\frac{dx_i(t)}{dt} = a_i x_i(t) + b_i u_i(t) + g_i v_i(t), \quad i = 1, 2, \cdots, K \tag{4.27}$$

where, $a_i = e^{-r_p \tau}(\beta R_i)$. Again, we represent the variables as follows.

$$v_i(t) \sim N(0, \tau_i^2(t)), \quad \tau_i^2(t) \sim IG(s_j.v_j), \quad v_i(t) \sim G^v \tag{4.28}$$

With respect to Equation (4.28), the number of clusters is unknown in advance. Alternatively, the number of clusters may differ depending on the point in time being considered. However, the upper limit of the number of clusters is fixed. Consequently, we could represent a linear equation (4.27).

5 Results

In this paper, we have modeled the theoretical background of a production process implemented in ASEP using division vectors. We constructed a dynamic model configuration of clusters including costs (division vector with type). Under the assumptions presented herein, the characteristics of the Dirkle distribution do not appear. These will be investigated in future work.

References

[1] Kenji Shirai and Yoshinori Amano: Analysis of Production Processes Using a Lead-Time Function; *International Journal of Innovative Computing, Information and Control*, Volume 12, No.1, pp.125-138, Februray, 2016

[2] T.Minemura and K.Nishinari: Improvement of production efficiency by changing number of lot in production line: *Reports of RIAM Symposium No.22AO-S8 Development in Nonlinear Wave: Phenomena and Modeling*, Proceedings of a symposium held at Chikushi Campus, Kyushu Universiy, Kasuga, Fukuoka, Japan, October, 2010

[3] Kenji Shirai and Yoshinori Amano: Production model using an asymmetric simple exclusion process; *International Journal of Innovative Computing, Information and Control*, Volume 14, No.1, pp.65–81, Februray, 2018

[4] K.Kamamura and H.Oh, etc: Higher-order approximation of the Burgers equation; *Research Institute for Mathematical Sciences, Kyoto University*, Vol.993, pp.35-45, 1997

[5] K.Shirai and Y.Amano: Analysis of fluctuations in production processes using Burgers equation; *International Journal of Innovative Computing, Information and Control*, Volume 12, Number 6 pp.1615–1628, October, 2016

[6] Kenji Shirai, Yoshinori Amano and Sigeru Omatu: Improving Throughput by Considering the Production Process; *International Journal of Innovative Computing, Information and Control*, Vol. 9, No.12, pp.4917-4930, December, 2013

[7] Kenji Shirai, Yoshinori Amano and Sigeru Omatu: Propagation of Working-Time Delay in Production; *International Journal of Innovative Computing, Information and Control*, Volume 10, Number 1, pp.169-182, February, 2014