State Estimation of Impregnated Sheet Substrates in a Drying Oven

Kenji Shirai*[1] Yoshinori Amano*[2]

Abstract

We have previously reported that a control system configuration can be designed for a drying oven using a mathematical state model, which we described using a transfer function with a quadratic time delay. Herein, a one-dimensional advection-diffusion equation (ODAE), in which the object model has a constant velocity v, defines the state of the drying oven in the target system.

The measurement of true state quantity is difficult for such a model. Therefore, we propose that the state estimation of such a model subjected to state-dependent noise is possible using optimal filter theory based on functional analysis. For state-independent noise, we can use the Kalman filter for conventional state estimation.

Keyword: stochastic partial diffusion equation, optimal filter, eigenvalue problem, drying oven, impregnating solvent

1 Introduction

In general, a machine produces sheet type films. Its function is to first impregnate a solvent, especially an organic solvent, into the films, and then control the thickness of the impregnated film using a heat source in the drying oven. Such a machine is called "impregnating machine," and it produces shaped insulator films. Some companies have developed various types of machines that process many films using various solvents in a conventional manner. As a result, many different types of films have been produced.

In previous studies related to impregnating machines, it is widely recognized that the impregnated solvent on sheet-type films gets dried while diffusing the heat in a drying oven, or the sheet-type films may themselves move during the process of receiving heat in the form of steam in the drying oven[1, 2, 3]. It is considered that the most important process units of the impregnating machine are the impregnating solvent and the drying unit.

Horiuchi et al. proposed a drying simulator which calculates the state variables that are related to the drying condition on impregnated films using the thermal diffusion in the drying

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oven[4]. We reported that the main state variables are given by an internal vapor pressure function to realize the design of a control system configuration[8, 9]. From the above description, the mathematical method uses a transfer function with a second-order time delay model as a more specific heat diffusion function in the drying oven[12]. Our previous study also provides a mathematical model in which the impregnated films move with a velocity “v” in a certain direction[6, 1]. The ordinary differential equation is given by the Lagrange Differential Operator (LDO). We also reported that the state of thermal diffusion to be discussed is defined by the one-dimensional advection diffusion equation (ODAE)[13].

Instead of focusing on the internal reaction when impregnating films, we were to design a dynamic state model on a control system. To do this, it is necessary to derive the mathematical model that describes the situation in which the solvent vapor is diffused by the vapor pressure unit.

In general, the Kalman filter is used in state estimation methods applied to finite-dimensional space. However, it is possible to take advantage of the optimal filter theory in infinite dimensional stochastic space[5].

In this study, we use the stochastic advection?diffusion model in infinite dimensional space. However, it is difficult for such a model to measure the true state quantity of the data. Therefore, we must utilize an optimal filter when receiving state-dependent noise in a drying oven[5, 13]. For state-independent noise, we can utilize the Kalman filter for conventional state estimation. In addition, we verify the state estimation using numerical results. To the best of our knowledge, previous studies have not clarified such an estimation problem extensively.

2 Basic mathematical model and definition of the physical quantities

With respect to a model that remains stationary for both the films and heat source in the drying oven shown in Fig.1, the physical quantities used in this section are described as follows[6, 7, 11].

Assuming that the vapor pressure is derived using the function of the moisture $q(t)$ in a

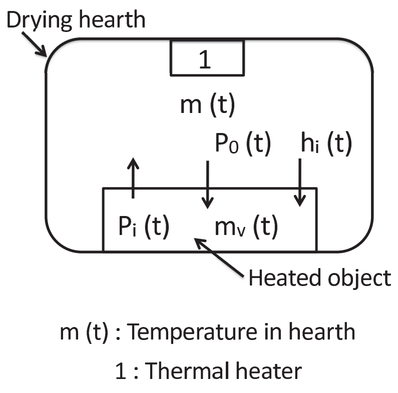


Fig. 1: Static model in the drying oven

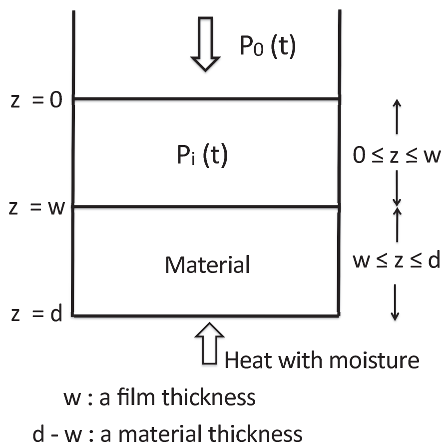


Fig. 2: Oven standard model

drying oven, the vapor pressure is derived by

$$P_0(t) = g_1[q(t),\ \ t \geq 0] \tag{2.1}$$

where, the moisture function $q(t)$ is defined as the following equation:

Definition 1 *Moisture function q(t)*

$$\begin{aligned} q(t) &= \int r(t)dt \\ &= \int g_2[P_i(t) - P_0(t)]dt, \quad t \geq 0 \end{aligned} \tag{2.2}$$

where $r(t)$ is the ratio of drying, $r(t) = g_2[P_i(t)\text{-}P_0(t)]$, $P_i(t)$ is the vapor pressure in the drying oven, and $P_0(t)$ is the external vapor pressure.

The vapor pressure derived by the gradient on the average moisture $m_v(t)$ on the films in the drying oven, is defined by

Definition 2 *Gradient on the average moisture $m_v(t)$ on the films*

$$P_i(t) = \left.\frac{\partial P_i}{\partial m_v}\right|_{m_v = m_v^*} \tag{2.3}$$

where, $m_v(t)$ is derived by

$$m_v(t) = \int H[h_i(t) - h_0(t)]dt, \quad t \geq 0 \tag{2.4}$$

where $h_i(t)$ is the amount of heat transfer and $h_0(t)$ is the heat consumption. Now, $h_i(t)$ is described by

$$h_i(t) = [g_3(m_v(t)) - g_4(m_v(t))]dt, \quad t \geq 0 \tag{2.5}$$

where, H is the function representing a physical constraint.

$g_k(\bullet)$, k = 1,2,3,4 in Eqs. (2.1)-(2.5) represents the constraint function on the each physical quantity.

In this case, the transfer function $W(s)$ in such a drying oven is derived by

$$W(s) = \frac{Q(s)}{M(s)} \tag{2.6}$$

where $Q(s)$ is the Laplace transform of the moisture spring from the solvent obtained using the heat source in the drying oven, and $M(s)$ is the Laplace transform of the heat source function (the temperature in drying oven)[13].

Such a thermal system model is generally derived as the time delay with a second-order system:

$$W(s) = K_G \frac{1}{a_3 + a_2 s + a_1 s^2} \tag{2.7}$$

where, let K_G, a_1, a_2, a_3 be a positive real number respectively.

From Eqs. (2.6)-(2.7), the following ODE is obtained formally by

$$a_1 \frac{d^2 q(t)}{dt^2} + a_2 \frac{dq(t)}{dt} + a_3 q(t) = K_G m(t) \tag{2.8}$$

where, the initial condition is $q(0) = q_0$[12].

The state variables on the object are considered as follows.
This is a mathematical model for which the films with the impregnated solvent represent the upper layer condition dried by the heat source under certain conditions (See Fig.2).

Since the moisture is approximately equal to the vapor pressure, we let the vapor pressure function be $C(x, t)$.

In addition, from Eq.(2.6), let the heat source (the temperature in the drying oven) again be $f(t)$. Then, $W(s)$ is derived by

$$W(s) = \frac{C(s)}{F(s)} \tag{2.9}$$

The model of Eq. (2.9) shows that the thickness of the impregnated solvent on films is proportional to the value of the moisture vapor pressure function based on time. In the case of a constraint condition such as heating the impregnating solvents, the thickness of the impregnated solvent on films decreases. However, the thickness of the films themselves maintains a constant value.

To mathematically model the continuous films, we assume that the films move with velocity v in the direction.

To describe the mathematical model, the Lagrange differential operator D/Dt is introduced by

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v \frac{\partial}{\partial x} \tag{2.10}$$

where x denotes the spatial variable in the direction of movement[13].

From Eqs. (2.8) and (2.10) can be rewritten by

$$a_1 \frac{D^2 q(t)}{dt^2} + a_2 \frac{Dq(t)}{dt} + a_3 q(t) = K_G m(t) \tag{2.11}$$

Similarly, Eq. (2.9) is derived by

$$\hat{a}_1 \frac{D^2 C(x,t)}{dt^2} + \hat{a}_2 \frac{DC(x,t)}{dt} + \hat{a}_3 q(t) = \hat{K}_G f(t) \tag{2.12}$$

For the state variable below, it is assumed that Eq. (2.12), which is described by the internal vapor pressure function $C(x, t)$ is the target model system.

From Eqs.(2.10) and (2.12) can be rewritten by

$$\hat{a}_1\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial x}\right)^2 C(x,t)+\hat{a}_2\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial x}\right)C(x,t)+\hat{a}_3 C(x,t)=Kf(x,t) \tag{2.13}$$

Assuming that the diffusion of moisture moves in one direction, by ignoring these terms $\left(\partial/\partial t\right)\cdot\left(\partial/\partial x\right)$ in Eqs. (2.13) and (2.14) is described as follows:

$$\begin{aligned}\frac{\partial C(x,t)}{\partial t}+v\frac{\partial C(x,t)}{\partial x}+C(x,t)&=Dv^2 f(x,t)\\&=D_c\frac{\partial^2 C(x,t)}{\partial x^2}+kf(x,t)\end{aligned} \tag{2.14}$$

Equation (2.14) represents ODAE, where D_c denotes the diffusion coefficient and $f(x, t)$ denotes the distribution function in the thermal diffusion state.

As the described above, it can be expressed using the PDEs in Eq. (2.14), in which the movement model of the continuous films has the state variable of the internal vapor pressure.

3 State estimation model subjected to state-dependent noise

Figure 3 shows the model on this equipment. From our previous study, Figure 4 shows the model of the impregnating solvent on the films[1]. In this study, we discuss the x-axis direction rather than the z-axis direction.

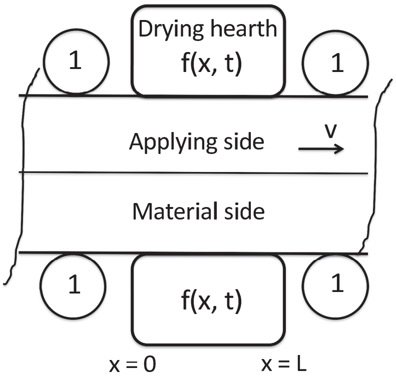
We describe the model subjected to state-dependent noise as follows.

$$\frac{\partial C(t,x)}{\partial t}=\mathscr{L}_x[C(t,x)]+B_d(t,x)C(t,x)W_d(t,x)+B_0(t,x)W_0(t,x) \tag{3.1}$$

$$C(0,x)=C_0(x),\quad C(t,x)\Big|_{x\in\partial D}=0 \tag{3.2}$$

where $L_x(\cdot)$ represents a diffusion operator.

$$\mathscr{L}_x[\cdot]=\left[v\frac{\partial}{\partial x}+D\frac{\partial^2}{\partial x^2}\right][\cdot] \tag{3.3}$$



1 : Drive shaft

Fig. 3: Actual plant system

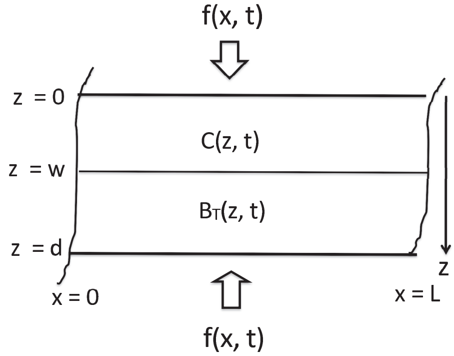


Fig. 4: Application of the model to material

Then the observation operator is as follows.

$$\partial Z(t,x) = \mathscr{H}_x[C(t,x)]\partial t + B_G(t,x)\partial W_e(t,x) \tag{3.4}$$

Here according to Hata's analytical method, we define a state variable $C(t, x)$ as follows[5].

Definition 3 *State variable* $C(t, x)$

$$C(t,x) \in C_1 = H^2(D) \tag{3.5}$$

$$W_d(t,x), \quad W_0(t,x), \quad W_e(t,x), \quad \in L^2(D) \tag{3.6}$$

Definition 4 *Observation variable* $Z(t, x)$

$$Z(t,x) \in H^2(D) \tag{3.7}$$

where $\hat{C}(t, x)$ is the estimated variable of $C(t, x)$.

Definition 5 *Estimated variable of* $\hat{C}(t, x)$

$$\hat{C}(t,x) \in H^2(D) \tag{3.8}$$

where, $\hat{C}(t, x)$ is derived as follows.

$$\hat{C}(t,x) = E[C(t,x)] \tag{3.9}$$

Moreover, when $X(t, x)$ is an element of the square integrable function space on D taking a scalar value, the inner product is as follows.

$$< X(t,x), X(t,s) >_{L^2(D)} = \int_D X^{'}(t,x) X(s,x) dx \tag{3.10}$$

Using this notation, Eq. (3.1) is as follows:

$$dC(t) = L(t)C(t)dt + B_d(t)C(t)dW_d(t) + B_0(t)dW_0(t) \tag{3.11}$$

where followings are satisfied.

$$\begin{aligned} E\{< X(t), W_d(t) >_{L^2(D)}\} &= 0 \\ E\{< X(t), W_0(t) >_{L^2(D)}\} &= 0 \\ E\{< W_d(t) \cdot W_d(s) >_{L^2(D)}\} &= Q_d \cdot \min(t,s) \\ E\{< W_0(t) \cdot W_0(s) >_{L^2(D)}\} &= Q_0 \cdot \min(t,s) \end{aligned} \tag{3.12}$$

Furthermore, the observation system is as follows:

$$dZ(t) = H(t)C(t)dt + G(t)dW_e(t) \tag{3.13}$$

where the followings are satisfied.

$$\begin{aligned} E\{< X(t), W_e(t) >_{L^2(D)}\} &= 0 \\ E\{< \hat{W}_e(t) \cdot W_e(s) >\} &= Q_e \cdot \min(t,s) \end{aligned} \tag{3.14}$$

The optimal filter for the estimated variable $\hat{C}$ (t, x) defined in Eq. (3.9) is constructed as follows:

$$\begin{aligned} \frac{\partial \hat{C}(t,x)}{\partial t} &= R_x[\hat{C}(t,x)] + \int_D S(t,x,x^{'}) \frac{\partial Z(t,x^{'})}{\partial t} dx^{'} \\ &= R(t,x)\hat{C}(t,x) + \int_D S(t,x,x^{'}) \frac{\partial Z(t,x^{'})}{\partial t} dx^{'} \end{aligned} \tag{3.15}$$

where $R(t, x)$ and $S(t, x, x')$ are determined to satisfy the following conditions:

$$E\left[< X(t,x), \{C(t,x) - \hat{C}(t,x) >_{L^2(D)}\right] = 0 \tag{3.16}$$

$$E\left[< X(t,x), \{\hat{C}(t,x) >^2_{L^2(D)}\right] \leq E\left[< X(t,x), \{C(t,x) - \hat{C}(t,x) >^2_{L^2(D)}\right] \tag{3.17}$$

According to Hata's analytical method and the above definitions and conditions, we obtain the following.

$$\hat{C}(t,x) \in H^2(D) \tag{3.18}$$

Then the optimal filter represents as follows:

$$d\hat{C}(t) = R(t)\hat{C}(t)dt + S(t)dZ(t) \tag{3.19}$$

where $\hat{C}(t)$ satisfies the following equations:

$$E\left[< X(t), \{\hat{C}(t) >_{L^2(D)} \right] = 0 \tag{3.20}$$

$$E\left[< X(t), \{\tilde{C}(t) >^2_{L^2(D)} \right] \leq E\left[< X(t), \{C(t) - \hat{C}(t) >^2_{L^2(D)} \right], \ ^\forall\tilde{C}(t) = C(t) - \hat{C}(t) \tag{3.21}$$

As a result, we can obtain the optimal filter and the equation to be satisfied as follows:

$$d\hat{C}(t) = L[\hat{C}(t)]dt + P(t)H^*(t)\left[G(t)\sqrt{Q_e}G(t)\sqrt{Q_e}\right]^{-1} \times \left[dZ(t) - H(t)\hat{C}(t)dt\right] \tag{3.22}$$

$$\begin{aligned}\frac{dP(t)}{dt} &= L(t)P(t) + P(t)L^*(t) + B_d(t)M_z(t)B_d^*(t) + B_0(t)\sqrt{Q_0} \times \sqrt{Q_0}B_0^*(t) \\ &\quad - P(t)H^*(t)\left\{G(t)\sqrt{Q_e} \times G(t)\sqrt{Q_e}\right\}^{-1} H(t)P(t)\end{aligned} \tag{3.23}$$

$$M_z(t) = E\left[C(t)\sqrt{Q_0} \times C(t)\sqrt{Q_0}\right] \tag{3.24}$$

where $M_z(t)$ satisfies as follows.

$$\begin{aligned}\frac{dM_z(t)}{dt} &= L(t)M_z(t) + M_z(t)L^*(t) + B_d(t)\sqrt{Q_d} \times M_z(t)\sqrt{Q_d}B_d^*(t) \\ &\quad + B_0(t)\left(Q_0Q_d\right)^{\frac{1}{2}} \times \left(Q_0Q_d\right)^{\frac{1}{2}} B_0^*(t)\end{aligned} \tag{3.25}$$

Further,

$$\hat{C}(0) = 0 \tag{3.26}$$

また,

$$\begin{aligned}P(t) &= E[\tilde{C}(t) \cdot \hat{C}(t)] \\ M_z(t) &= E[C(t) \cdot Q_d(t)C(t)]\end{aligned}$$

Finally, from Eq. (3.19), the optimal filter gain is obtained by Eq. (??).

$$S(t) = \left(G(t)Q_eG(t)\right)^{-1} H^*(t)P(t) \tag{3.27}$$

$$R(t) = L(t) - S(t)H(t) \tag{3.28}$$

4 Eigenvalue problem subjected to state-dependent noise

Here we focus on an eigenvalue problem for Eq. (3.1), which can be described as follows.

$$\frac{\partial C(t,x)}{\partial t} = \mathscr{L}_x C(t,x) + \sigma_d C(t,x) B_d(t,x) + \sigma_0 B_0(t,x) \tag{4.1}$$

Definition 6 *Eigenvalue problem*

$$\mathscr{L}_x(t,x) = \lambda \varphi(t,x) \tag{4.2}$$

where $\varphi(t, x)$ represents an eigenfunction and λ is an eigenvalue. Both these terms are discrete eigenvalues.

Using Green's theorem, we transform Eqs. (4.1) and (4.2) to obtain the following.

$$\int_{\partial D} \varphi \frac{\partial C}{\partial t} dx - \int_{\partial D} C\mathscr{L}(\varphi) dx = \int_{\partial D} \left[\left\{ \varphi \frac{\partial C}{\partial x} - C \frac{\partial \varphi}{\partial x} \right\} \right] dx + \int_D \sigma_d \varphi \cdot C B_d(t,x) dx + \int_D \sigma_0 \varphi B_0(t,x) dx \tag{4.3}$$

We rewrite Eq. (4.3) and obtain the following.

$$\int_{\partial D} \left[\left\{ \varphi \frac{\partial C}{\partial x} - C \frac{\partial \varphi}{\partial x} \right\} \right] dx = \int_{\partial D} [\varphi P_x(C) - C P_x(\varphi)] dt + \sigma_d \int_D C(t,x) \varphi(t,x) B_d(t,x) dx + \sigma_0 \int_D \varphi(t,x) B_0(t,x) dx \tag{4.4}$$

Here as the first term on the right hand side in Eq. (4.4) represents zero, as indicated in Eq. (3.2), the equation can be rewritten as follows.

$$\int_{\partial D} \left[\left\{ \varphi \frac{\partial C}{\partial x} - C \frac{\partial \varphi}{\partial x} \right\} \right] dx = \sigma_d \int_D C(t,x) \varphi(t,x) B_d(t,x) dx + \sigma_0 \int_D \varphi(t,x) B_0(t,x) dx \tag{4.5}$$

From Eq. (4.5), we can obtain as follows.

$$\frac{dC(t)}{dt} = \lambda C(t) + \sigma_d C(t) B_d(t) + \sigma_0 B_0(t) \tag{4.6}$$

where Eq. (4.6) satisfies as follows.

$$C(t,x) = \int_D C(t,x) \varphi(t,x) dx \tag{4.7}$$

$$\sigma_D C(t) W_d(t) = \sigma_d \int_D C(t,x) \varphi(t,x) W_d(t,x) dx \tag{4.8}$$

$$\sigma_0 W_0(t) = \sigma_0 \int_D \varphi(t,x) W_0(t,x) dx \tag{4.9}$$

Therefore we can obtain as follows.

$$dC(t) = \lambda C(t)dt + \sigma_d C(t)dW_d(t) + \sigma_0 dW_0(t) \tag{4.10}$$

where $C(t) \in U \in H^2(D)$.
In the same manner, we rewrite Eq. (4.10) to obtain the following.

$$dZ(t) = hC(t)dt + \sigma_G dW_e(T) \tag{4.11}$$

Further Eq. (4.11) satisfies Eqs. (3.12), (3.14).
Then, Eq. (3.22) is described as follows.

$$d\hat{C}(t) = \lambda \hat{C}(t) + P(t)h\left(\frac{1}{\sigma_G^2 Q_e}\right) \times [dZ(t) - h\hat{C}(t)dt] \tag{4.12}$$

Eqs. (3.23)-(??) are obtained as follows.

$$\frac{dP(t)}{dt} = 2\lambda P(t) + \sigma_d^2 M_z(t) + \sigma_0^2 Q_0 - h^2 P^2(t)\left(\frac{1}{\sigma_G^2 Q_e}\right) \tag{4.13}$$

$$\frac{dM_z(t)}{dt} = 2\lambda M_z(t) + \sigma_d^2 Q_d M_z(t) + \sigma_0^2 Q_0 Q_d \tag{4.14}$$

where $P(t)$ and $M_z(t)$ are derived as follows.

$$P(t) = E[\tilde{C}(t) \cdot \hat{C}(t)] \tag{4.15}$$

$$M_z(t) = E[C(t) \cdot Q_d \cdot C(t)] \tag{4.16}$$

As described above, according to Hata's analytical method, we can construct an optimal filter for a convection?diffusion system that receives state-dependent noise on the research target as Eqs. (4.12)-(4.16).

5 State estimation model subject to state-independent noise

Here we focus on the following target model.

$$\frac{\partial C(t,x)}{\partial t} + v\frac{\partial C(t,x)}{\partial x} = D\frac{\partial^2 C(t,x)}{\partial x^2} + \sigma B(t,x) \tag{5.1}$$

$$C(0,x) = C_0(x), \ \ x \in D, \ \ t \in R_+ \tag{5.2}$$

$$\left.\frac{\partial C}{\partial x}\right|_{x \in \partial D} = 0 \tag{5.3}$$

The discrete eigenvalue λ_i of the model in Eq. (5.1) on $C(t) \in H^2(D)$ satisfies the following definition.

Definition 7 *Model being used as a discrete eigenvalue*

$$dC_i(t) = \lambda_i C_i(t) + \sigma dW_d(t), \quad i = 1, 2, \cdots \tag{5.4}$$

Furthermore, the observation system is as follows.

Definition 8 *Observation system model*

$$dZ_i(t) = \varphi_i(x_m) C_i(t) dt + \sigma_e dW_e(t), \quad i = 1, 2, \cdots \tag{5.5}$$

Using the Kalman filter, the estimated variable $\hat{C}(t)$ and secondary moment of $C(t)$, respectively, are defined as follows.

Definition 9 *Estimated variable $\hat{C}(t)$ and secondary moment U(t) of C(t)*

$$\hat{C}(t) = E_t[C(t)] \tag{5.6}$$

$$U(t) = E_t[C(t) \cdot C(t)] \tag{5.7}$$

Then, from the analysis of Eqs. (4.1)--(4.11), Eqs. (5.4)--(5.5) can be obtained for the model of Eqs. (5.1)--(5.3).

6 Numerical Example

The calculated results are shown in Fig.5--Fig.13. As shown in Fig.10, in this system, the first eigenvalue of the model system reflects a strong effect, which affects a wide range of time over the spatial domain. As shown in Fig.13, the outlet-side neighborhood is close to the model state in the case of an advection.diffusion system. Fig.14 represents the processes of both system model and estimation model. Then Fig.15 represents the processes of both system model and observation model. Therefore, for simplicity, we calculated only the first eigenvalue for diffusion coefficient $\mu = 0.5$.

$$dC(t) = 1.08 \cdot C(t) dt + \sigma dW(t) \tag{6.1}$$

For $\sigma = 0.3$, we constructed an observation system using $\varphi(x_m) = 1.1$ and $\sigma e = 0.1$. In this case, we estimated the model represented by Eq. (5.1) successfully.

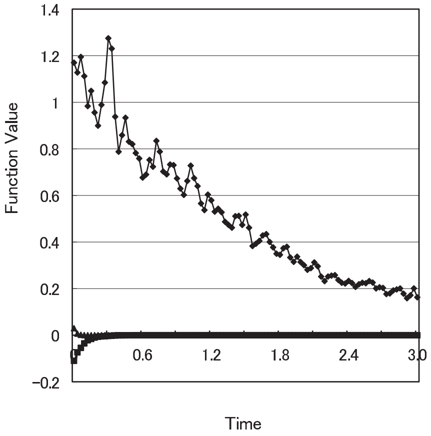


Fig. 5: Solution of a diffusion-type stochastic partial differential equation (μ = 0.5)

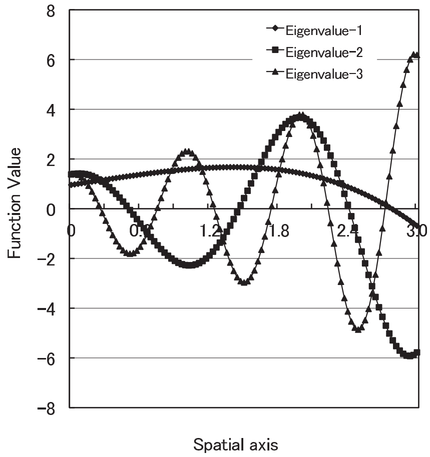


Fig. 6: Three eigenfunctions in a diffusion process

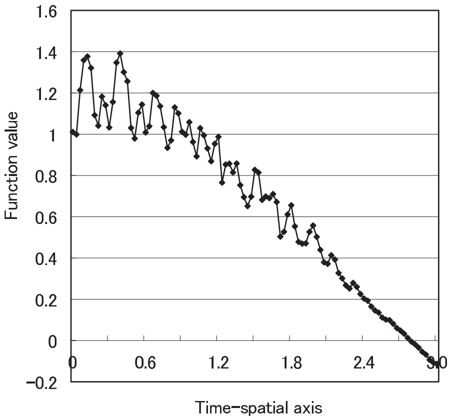


Fig. 7: Function value versus three eigenvalues

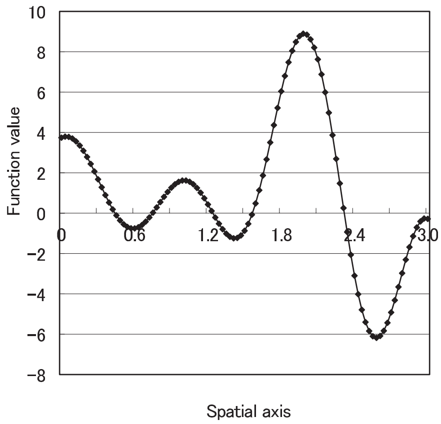


Fig. 8: Total function value versus sum of three eigenvalue function

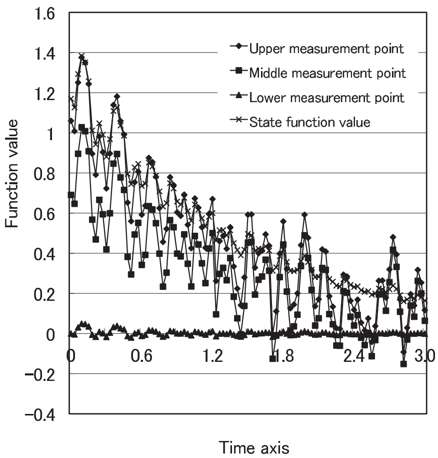


Fig. 9: State variable and measurement data versus

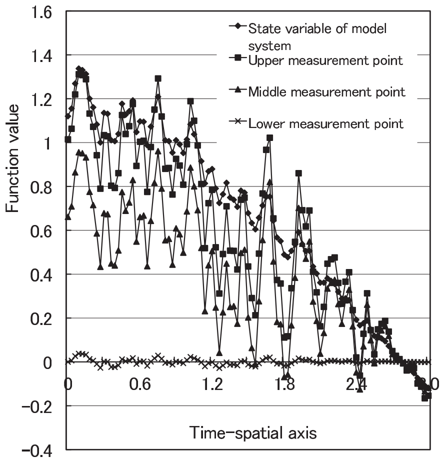


Fig. 10: State variable and measurement data versus time-space

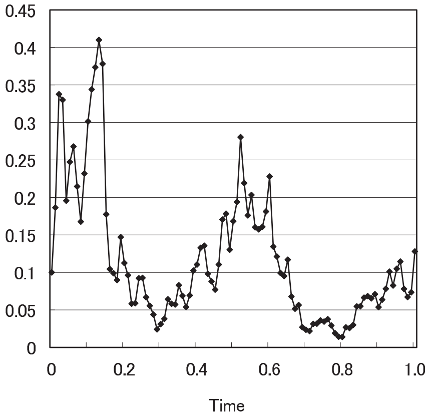


Fig. 11: Solution process of stochastic partial differential model versus time

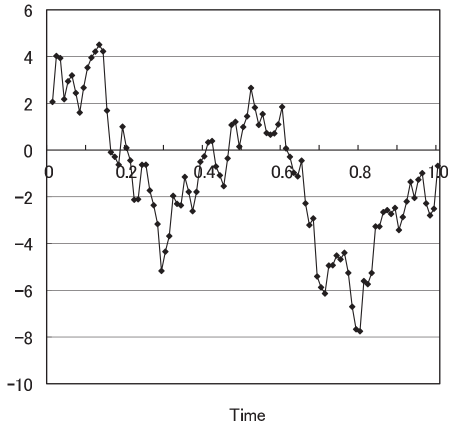


Fig. 12: Wiener process

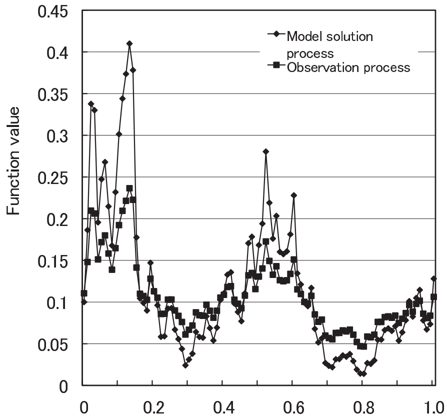


Fig. 13: System model and observation processes

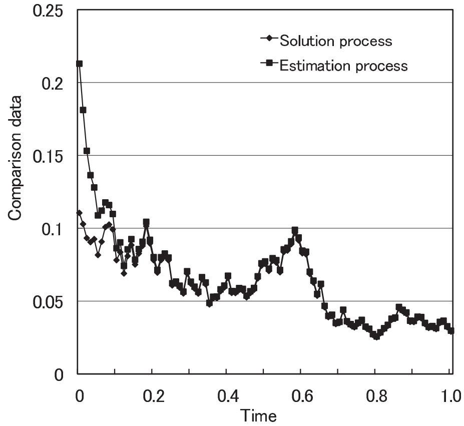


Fig. 14: Estimation process of the system model by Kalman filter

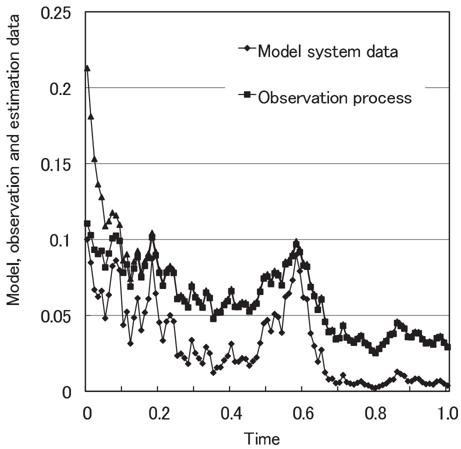


Fig. 15: System model, observed and estimation data

7 Results

In this study, we utilized a transfer function derived from a advection-type stochastic partial differential equation using a dynamic model. In addition, the transfer function can be obtained from a stochastic partial differential equation using a static model. Using the transfer function, we proposed a stochastic partial differential equation model that is subject to both the presence or absence of state-dependent noise relatively easily.

Using an optimal filter for the estimation problem that is subject to the state-dependent noise system, we could easily estimate state temperature in a drying oven. In addition, for the stateindependent noise system, we used the Kalman filter for conventional estimation. There are still many problems that require identification of various parameters and functions before practical application becomes feasible. However, our approach is useful from the viewpoint of the application of control theory, which is derived from a rich partial differential model.

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