A dynamic span model and associated control strategy for roll transport systems for sheet-type materials (Part I)

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Abstract

In this study, we propose a mathematical model of the tension occurring at stands between rolls in a roll transport system, and then we propose an optimal control system. The model is a distributed parameter model described by a partial differential equation (PDE). The PDE is derived from particular equilibrium conditions, which balance spatial forward deviation at a stand between rolls and temporal variation of tension. To build a realistic system, we utilized an optimal regulator theory derived by converting a PDE to an ordinary differential equation in a functional space. As a result, the proposed system is highly feasible.

Keyword: partial diffusion equation, tension, optimal control, distributed parameter system, sheet materials.

1 Introduction

We previously reported on the use of a mathematical state model to design a control system configuration for a drying oven. The model is described using a transfer function with a quadratic time delay[1, 2]. The state of the drying oven in the control system is defined by a one-dimensional advection?diffusion equation (ODE) in which the object model has a constant velocity of v. However, developing a quantifiable state estimate is difficult with such a model. Therefore, we propose to use optimal filter theory based on functional analysis to estimate the state of such a model when subjected to state-dependent noise. For state-independent noise, we can use the Kalman filter for conventional state estimation.

In our previous Bulletin[1, 2], we reported that when materials on a sheet in a drying oven moved, we applied a vapor pressure propagation model to the solvent contained in substrates in an effort to solve the state estimation problem. By applying this model, it is possible to design an optimal control system

Our findings related to transportation of sheet substrates were recently applied under a manufacturing context, to a roll transport system for textile processing machinery[3]. The model has also been used recently for a sheet transport/thin film transport system and with various chemically processed substrates such as a nonmoving fabric transport system.

Roll transport systems are important mechanical elements of drive systems in transport drive systems for sheet substrates[8]. The underlying principal of these systems for transporting substrates is to apply a vertical load of a cylindrical roll and a frictional force between substrates and rolls, as illustrated in Fig. 1. It is then possible to change the angle of sheet substrate transport by connecting a motor and using that motor to forcibly rotate a drive roll in a given direction (see Fig. 2).

Generally, manufacturing equipment lines are long, and the drive rolls are utilized with multiple pieces of equipment. In many cases, a drive roll is used with well over twenty pieces of equipment. The equipment that supplies substrates to these systems, known as an unwinding device, is usually installed at the inlet side. Then, sheet-type base substrates are fed using a driving

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roll product wound into a roll. Then, the device, which is called a winder for winding the processed product, is in some cases installed at an outlet. Obviously, other devices are in some cases installed in certain locations.

The first system control problem in roll transport systems is process control in various processing devices and the second is control of the tension generated in the transport middle base of the sheet substrate; the latter is a major issue. In this study, we considered a dynamic model to control tension in materials during transportation. Our study is based on metal rolling process theory[4, 5]. We then applied appropriate parameters in a tandem mill control system [Remark 4] to resolve the material tension problem that occurs when dealing with substrate films[9, 10, 11, 12, 13]. Moreover, using the model, we were able to study another control problem; we applied a boundary control function to study optimal speed tracking[16, 17].

2 Tension development model

2.1 Factors of tension development

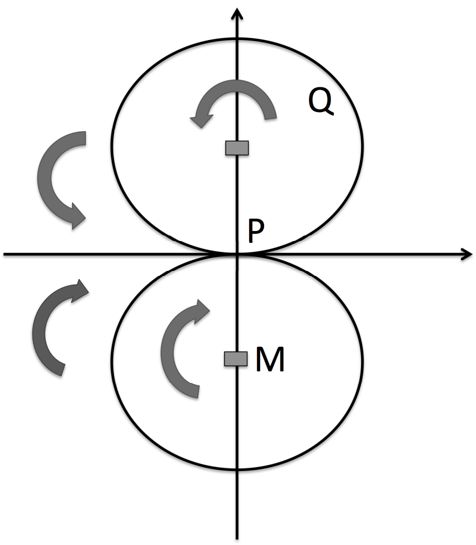

Fig. 1: Unit model (1) of a roll transport system

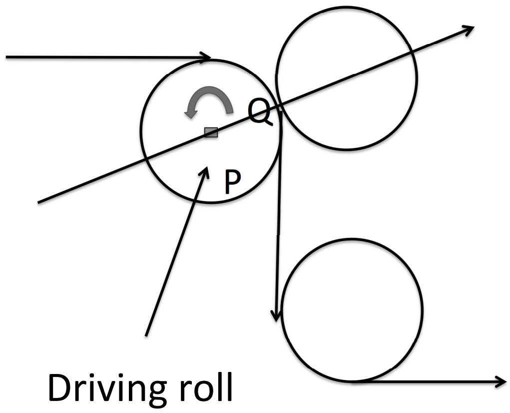

Fig. 2: Unit model (2) of a roll transport system

Table. 1: Physical meaning of each symbol

κ	Friction coefficient
Q	Back tension
P	Vertical load
V	The average speed of the drive roll and the substrate [m/min]
v_s	Speed difference between the drive roll and the substrate [m/min]
v_1	Circumferential speed of the drive roll [m/min]
v_2	Conveying speed of the substrate [m/min]

We obtain[14]:

$$\frac{v_s}{V} = \frac{v_1 - v_2}{\frac{1}{2}(v_1 + v_2)} \approx f\Big(\frac{Q}{\kappa P}\Big) \tag{2.1}$$

where, Eqn.(2.1) is called as a slip characteristics of macro.

Now, we describe more detail as follows: From $v_s = v_1 - v_2$,

$$v_s = \delta v_1 - \delta v_2 + V\Big(\frac{\partial u_1}{\partial x} - \frac{\partial u_2}{\partial x}\Big) \tag{2.2}$$

From Eqn.(2.2),

$$\frac{v_s}{V} = \Big(\frac{\delta v_1 - \delta v_2}{V}\Big) + h(x) \tag{2.3}$$

where, $h(x)$ is a tension on the surface and a function of pressure distribution between rolls.

"Slipping" is the difference between the circumferential speed of the drive roll and the transportation speed of the materials. It is dependent on dynamic characteristics including pull force and pressure distribution that exist on the surface between the rolls and between the materials. When slipping occurs on the surface of a roll, materials between rolls change their transportation direction, which creates tension. As can be seen in Fig. 5, the tension on a stand between rolls originates in a length between the rolls because of the dynamic characteristics of materials and the speed difference between the rolls. We can use Young's modulus to quantify these factors[6].

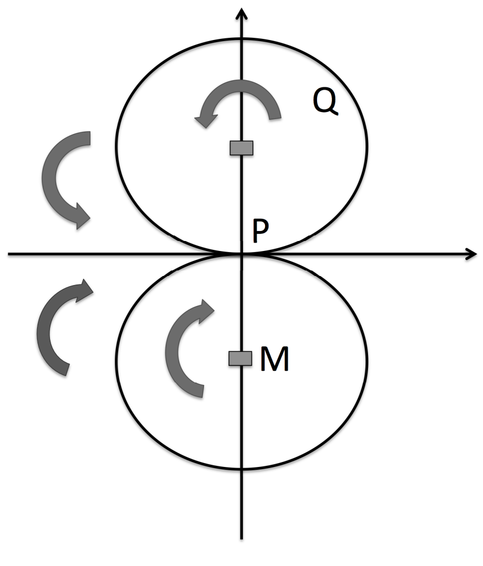

Fig. 3: General idea of slip phenomenon

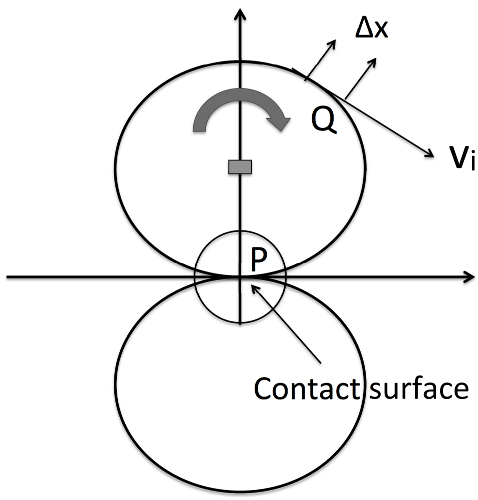

Fig. 4: Detail figure of slip phenomenon

Then, with respect to a drive roll same as Eqn.(2.1),

$$\frac{v_s}{V} = \Big(\frac{\delta v_1 - \delta v_2}{\frac{1}{2}(v_{b1} + v_{b2})}\Big) \approx f\Big[\frac{Q_1}{\kappa_1 P_1}\Big] \tag{2.4}$$

$$C(s) \approx \frac{E}{I}\Big[v_i(s) - v_{i+1}(s)\Big] \tag{2.5}$$

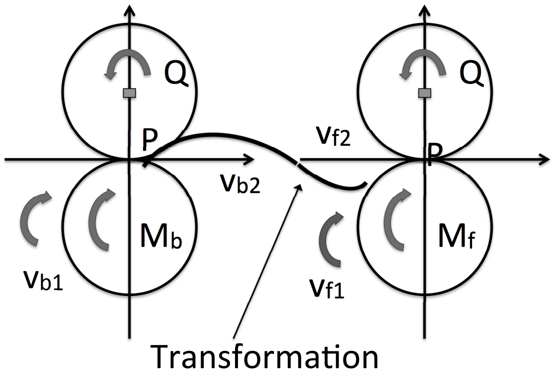

Fig. 5: Materials transformation model in the stand between rolls

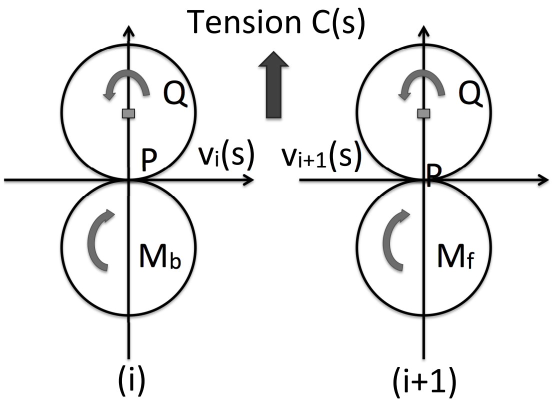

Fig. 6: Tension outbreak model for stand between rolls

where E is a Young's modulus, represent hereafter.

As described above, the tension between the rolls and between the materials on a stand originates in the difference between the speeds of the rolls. Furthermore, slipping on the contact surface between materials and rolls creates tension as material volume shifts between forward rolls and receiving rolls. Now, slipping occurs on an independent roll, and is determined by a friction force and drive rolls among vertical loads of rolls and materials. Then, we consider the dynamic parameters of tension that originates in the materials using Young's modulus. In considering tension on a stand between rolls and across a length between stands, we consider whether the transformation that occurs is a slackening or a heightening of tension[7]. We can then construct a distributed parameter system of tension and consider a lumped parameter system, which we will report next year in a manuscript.

Young's modulus is a proportional constant data between a distortion of coaxial direction and a stress in an elastic range established by Hooke's law. In Fig. 7 the value of Young's modulus is given by about $\sigma(stress)/\varepsilon(Strain)$[18].

2.2 Distributed parameters system

Fig. 8 shows a whole conceptual model and Fig. 9 shows a local model describing slackening and tension at $(x, x+\Delta x)$.

Now, From Fig.9,

$$[J(x,t) - J(x+\Delta x,t)] \approx [C(x,t+\Delta t) - C(x,t)] \tag{2.6}$$

That is, Eqn.(2.6) represents a slipping between materials and rolls, and speed fluctuation of drive rolls[15].

$$[a\ \ sheet\ \ forward\ \ change] \approx [a\ \ tension\ \ deviation\ \ on\ \ stand\ \ between\ \ rolls] \tag{2.7}$$

From Eqs.(2.6) and (2.7),

$$[J(x,t) - J(x+\Delta x,t)]E\cdot\Delta t \approx [C(x,t+\Delta t) - C(x,t)]L\cdot\Delta x \tag{2.8}$$

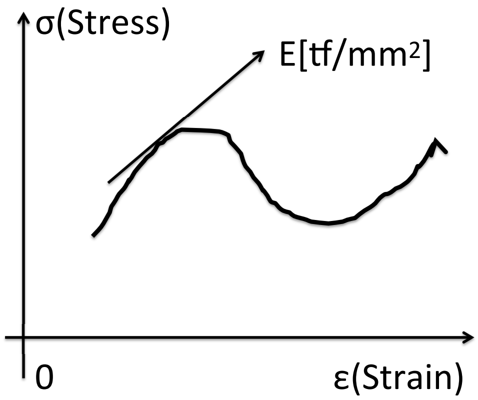

Fig. 7: Concept of Young's modulus

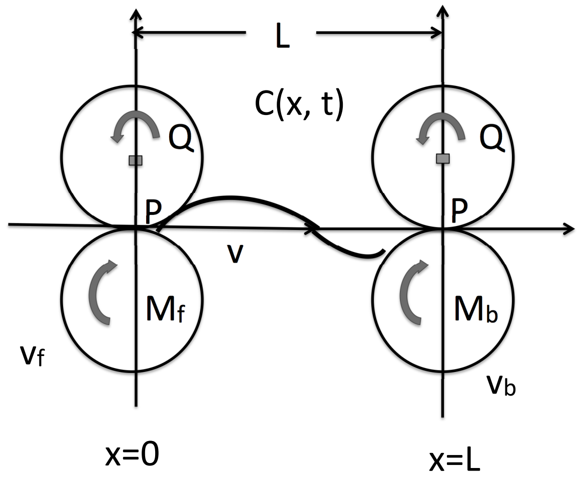

Fig. 8: Tension model on stands between rolls

Table. 2: Physical meaning of each symbol

M_f	Front driving motor
v_f	Front driving motor rotation speed [m/min]
M_b	Rear driving motor
v_b	Rear driving motor rotation speed [m/min]
v	Sheet transportation speed [m/min]
L	Length between rolls [m]
E	Young's modulus [tf/mm^2] or [GPa]
$C(x,t)$	Distributed tension function
x	Space variable
t	Time variable

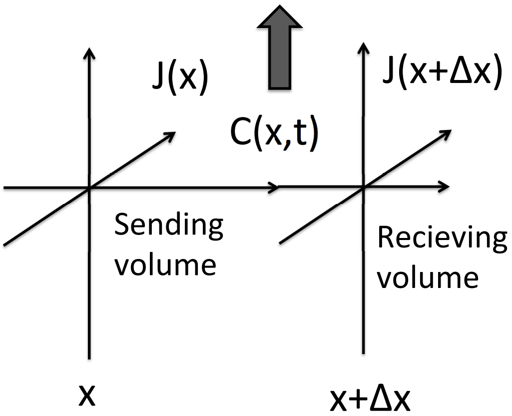

Fig. 9: Local tension model on a stand between rolls

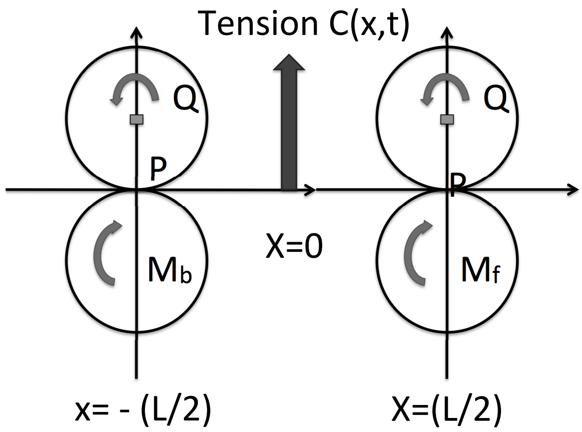

Fig. 10: Tension originating model on a stand between rolls

Eqn.(2.8) represents deviation of sending volumes and receiving volumes, that is, tension that originates in a slackening or heightening of tension.

From Eqn.(2.8),

$$-\frac{\Delta J}{\Delta x}\cdot E=\frac{\Delta C}{\Delta t}\cdot L \tag{2.9}$$

Here, if $\Delta x\rightarrow 0$, $\Delta t\rightarrow 0$, Eqn.(2.9):

$$-\frac{\partial J}{\partial x}\left(\frac{E}{L}\right)=\frac{\partial C}{\partial t} \tag{2.10}$$

Then, we consider that materials flow causes tension deviation against forward transportation.

$$J=-\frac{\partial C}{\partial x} \tag{2.11}$$

From Eqs.(2.10) and (2.11),

$$\frac{\partial C}{\partial t}=\left(\frac{E}{L}\right)\frac{\partial^2 C}{\partial x^2} \tag{2.12}$$

Thus, Eqn.(2.12) is a one-dimensional diffusion equation that represents diffusional transportation in both directions from an origin point that is the coordinate axes, and in a case of no external force. A particular solution of PDE for diffusional transportation is obtained:

$$C(x,t)=\frac{1}{\sqrt{4\pi Dt}}\exp\left\{-\frac{x^2}{4Dt}\right\} \tag{2.13}$$

where, $D=E/L$.

To solve the equation, an appropriate initial condition and boundary condition are needed. For example, the boundary condition is at the roll transportation point[20, 21].

$$\left.\frac{\partial C}{\partial x}\right|_{x=\partial\Omega_1} = 0, \quad \left.\frac{\partial C}{\partial x}\right|_{x=\partial\Omega_2} = 0 \tag{2.14}$$

Eqn. (2.14) shows that tension fluctuation can be ignored at a contact point between materials and rolls. For the case of transportation of materials with speed v, a tension function $C(x,t)$ is described:

$$\frac{\partial C(x,t)}{\partial t} + v\frac{\partial C(x,t)}{\partial x} = D\frac{\partial^2 C}{\partial x^2} \tag{2.15}$$

, where $D = (E/L)$, $x \in [0,L]$, $t \in [0,T]$, L is a stand length between rolls and E is Young's modulus.

Eqn. (2.15) can be modeled as Fig. 10. Fig. 11 shows a local model at an origin point of tension.

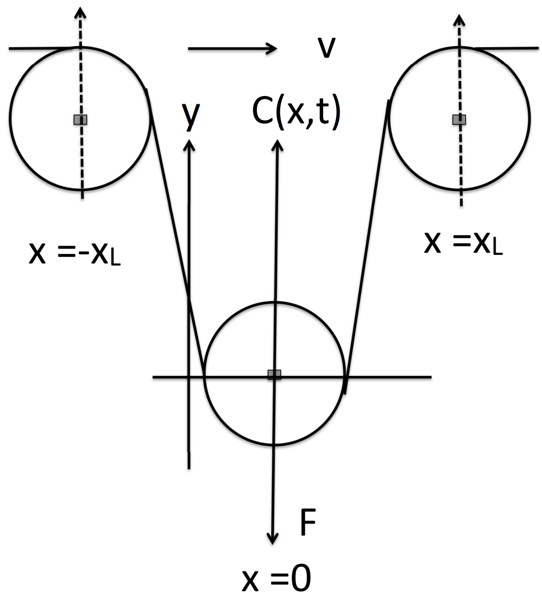

Fig. 11: The local model of Fig.10

Now, a local model is obtained from Fig.11:

$$\frac{\partial C(x,t)}{\partial t} = D\frac{\partial^2 C}{\partial x^2}, \tag{2.16}$$

$$Initial\ \ condition: \ C_0(x,0) = C_0(x) \tag{2.17}$$

$$Boundary\ \ condition: \ \left.\frac{\partial C(x,t)}{\partial x}\right|_{x=-X_L} = \left.\frac{\partial C(x,t)}{\partial x}\right|_{x=+X_L} = 0 \tag{2.18}$$

where $D = (E/L)$, $x \in [-x_L, +x_L]$, $t \in [0,T]$.

The rolls used between drive rolls are called “ dancer rolls ” (see Fig. 11). Tension at the dancer roll can be obtained as a fixed tension in despite position fluctuation, by adding force with a fixed direction from a spring. This location is used to “pick-up” excess tension. In other words, the dancer roll is a piece of tension equipment that can be moved toward the y-axes and installed at an appropriate place on the stand to absorb material slack.

The solution of Eqn. (2.13) shows that if materials have a constant Young' modulus E, then their diffusion coefficient D is in inverse proportion to length L. Accordingly, as $L \to$ *small*, let $D \to$ *large*, a tension appears, with break-up around a certain central point. Conversely, as $L \to$ *large*, let $D \to$ *small*, a tension appears, with centralization around a certain central point, and gets small skirts.

With Eqn. (2.15), the tension distribution, which changes the transport direction widely, can be obtained. We can obtain Figs. 12 to 15 using Eqn. (2.15) with appropriate parameters.

$$C(x,t) = \frac{1}{\sqrt{4\pi Dt}} \exp\left\{-\frac{(x-vt)^2}{4Dt}\right\} \tag{2.19}$$

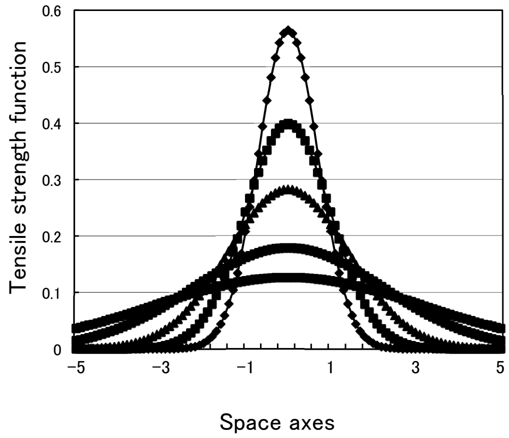

Fig. 12: Distributed tension values(Diffusion coefficient = 0.5)

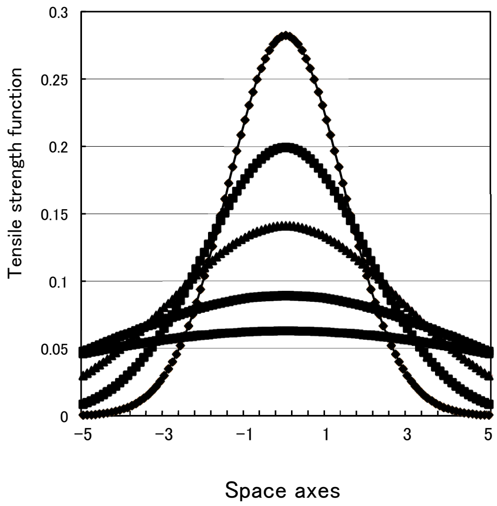

Fig. 13: Distributed tension values(Diffusion coefficient = 2)

Figs. 12 and 13 show distributed tension values with diffusion coefficients of 0.5 and 1.0, respectively. Figs. 14 and 15show a diffusion convex?type equation and distributed tension function with moving speeds of 0.5 and 1.0, respectively.

3 Tension control system design

We modeled a tension control system design for a realistic environment, as shown in Fig. 16.

Table. 3: Physical meaning of each symbol

$v_f(t)$	materials forward speed 0.5zw [m/min]
$v_b(t)$	a tensile speed of a standard material [m/min]

Here, L, E, $C(x,t)$, x and t are same thing in the previous section.

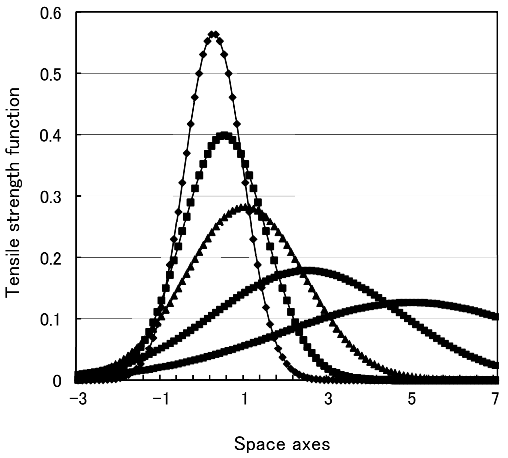

Fig. 14: Distributed tension values(Moving speed = 0.5)

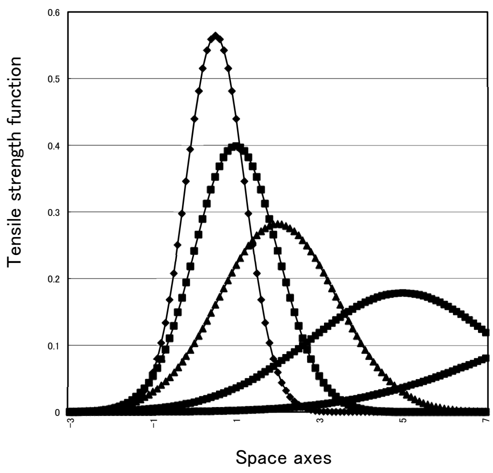

Fig. 15: Distributed tension values(Moving speed = 1)

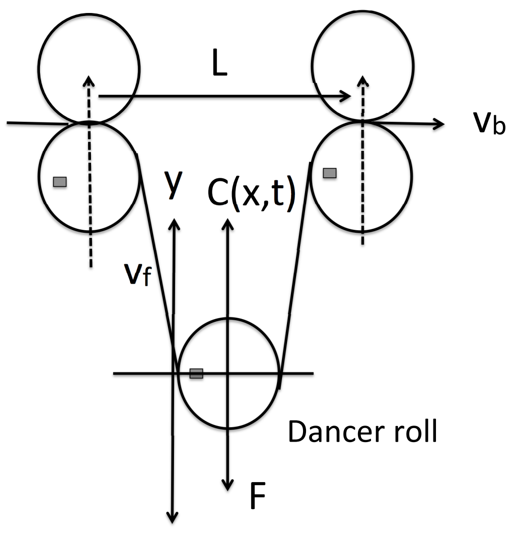

Fig. 16: The local model of control system

Definition 1 *Control system specification A transportation speed makes tracking to a standard tensile speed $v_f(t)$ during transportation between rolls. Then, $v_f(t)$ is controlled by tension $C(x,t)$, which maintains a zero equilibrium. Here, $v_f(t)$ is a control parameter at the boundary condition.*

From above description, the target system model is described:

$$
\begin{aligned}
\frac{\partial C(x,t)}{\partial t} &= D\frac{\partial^2 C}{\partial x^2} + b\delta(x-x^{'})v_f(t) + b\delta(x-x^{'})v_0 \\
&= D\frac{\partial^2 C}{\partial x^2} + b(x)f(x,t) \qquad (3.1) \\
f(x,t) &= \delta(x-x^{'})v_f(t) + \delta(x-x^{'})v_b(t) \qquad (3.2)
\end{aligned}
$$

where, $b > 0$ is a constant and $\delta(x)$ is a delta function.

Accordingly,

Definition 2 *Evaluation functional*

$$
J = \int_{x\in\Omega, t\in R} [PC^2(x,t) + q(x)g^2(x,t)]dxdt \qquad (3.3)
$$

where, P and $q(x)$ are a weight parameter and function, $P > 0$ and $q(x) > 0$.

A function $g(x,t)$ is:

$$
g(x,t) = b(x)f(x,t) \qquad (3.4)
$$

While optimal control is determined by a strict partial differential equation (PDE):

$$
\begin{aligned}
g^*(x,t) &= -K(x,t)C(x,t) \qquad (3.5) \\
K(x,t) &= q^{-1}(x)b(x)P(x,t) \qquad (3.6)
\end{aligned}
$$

where $K(x,t)$ is an optimal feedback function, and $P(x,t)$ satisfies the following Riccati-type PDE[22]:

$$
\frac{\partial P(x,t)}{\partial t} = 2\mathscr{L}\Big[P(x,t)\Big] - q(x)b(x)P^2(x,t) + P^2 \qquad (3.7)
$$

where,

$$
\begin{aligned}
\mathscr{L}(\bullet) &= D\frac{\partial}{\partial x^2} \qquad (3.8) \\
P(x,T) &= 0 \qquad (3.9)
\end{aligned}
$$

From the above description, a strict solution is obtained. However, as we are considering an actual engineering problem, it is important to propose a realistic model.

Thus, we consider a next model:

Definition 3 *Actual model*

$$
\begin{aligned}
\frac{\partial C(x,t)}{\partial t} &= D\frac{\partial^2 C(x,t)}{\partial x^2} \qquad (3.10) \\
\frac{\partial C(x,t)}{\partial x}\Big|_{x=0} &= r_1 v_f(t) \qquad (3.11) \\
\frac{\partial C(x,t)}{\partial x}\Big|_{x=L_1} &= r_2 v_b(t) \qquad (3.12) \\
C(x,0) &= C_0(x) \qquad (3.13)
\end{aligned}
$$

where $D = (D/L)$, $x \in [0, L]$, $t \in [0, T]$, $r_1 > 0$ and $r_2 > 0$.

Eqs.(3.10) to (3.13) are the actual model, which has a boundary control function.

Generally, when let the x's range to $x \in \Omega$, let its boundary to $x \in \partial\Omega$.

Here, Green's theorem is used to Eqs.(3.10) to (3.13) and we obtain:

$$\int_\Omega \varphi \frac{\partial C}{\partial t} - \int_\Omega C\Big(D\frac{\partial^2 \varphi}{\partial x^2}\Big)dx = \int_{\partial\Omega}\Big\{\Big[\varphi\frac{\partial C}{\partial t} - C\frac{\partial \varphi}{\partial x}\Big]\Big\}dx \tag{3.14}$$

We apply an eigenvalue problem to the terms in the left of Eqn.(3.14):

$$\int_\Omega \Big\{\Big(\varphi\frac{\partial C}{\partial t} - \int_\Omega \Big(D\frac{\partial^2 \varphi}{\partial x^2}\Big)\cdot C\Big)\Big\}dx \tag{3.15}$$

We rewrite Eqn.(3.15):

$$\mathscr{L}(\varphi_i) = \lambda_i \varphi_i \tag{3.16}$$

where, $i = 1, 2, \cdots$, φ_i is a eigen-function, λ_i is a eigenvalue.

From Eqn.(3.16), we obtain:

$$\int_\Omega \Big[\frac{\partial C}{\partial t} - \lambda C\Big]\varphi dx = \int_{\partial\Omega}\Big(\varphi\frac{\partial C}{\partial x} - C\frac{\partial \varphi}{\partial x}\Big)dx \tag{3.17}$$

where, let the boundary condition of Eqn.(3.17) to $(\partial\varphi/\partial x)\Big|_{x\in\Omega} = 0$.

From Eqn.(3.17) with boundary condition, we obtain[15]:

$$\frac{dC_i(t)}{dt} = \lambda_i C_i(t) + r_1\varphi_i(0)v_f(t) - r_2\varphi_i(L_1)v_b \tag{3.18}$$

where,

$$C(x,t) = \sum_{i=1} C_i(t)\varphi_i(x) \tag{3.19}$$

$$C_i(t) = \int_\Omega C(x,t)\varphi_i(x)dx \tag{3.20}$$

where $i = 1, 2, \cdots$

Accordingly, Eqn.(3.18) represents a system model, which considers a control function $v_f(t)$ at the boundary $x = 0$ in a functional space $H^2(\Omega)$.

Here, the subscript i is omitted hereafter, then we obtain:

$$\frac{dC(t)}{dt} = \lambda C(t) + bf(t) \tag{3.21}$$

where,

$$bf(t) = r_1\varphi(0)v_f(t) - r_2\varphi(L_1)v_b \tag{3.22}$$

According to Eqn.(3.20), and from $C(t) \in H^2(\Omega)$:

$$C(t) = \int_\Omega C(x,t)\varphi(x)dx \tag{3.23}$$

According to Eqn.(3.3), A evaluation functionals is:

Assumption 1 *Evaluation functionals*

$$J = \int_0^T [PC^2(t) + qf^2(t)]dt \tag{3.24}$$

Then, according to Eqs.(3.5) and (3.6),

$$f(t) = -K(t)C(t) \tag{3.25}$$

where, $K(t) = q^{-1}bP(t)$ is a optimal feedback function.

Moreover, according to Eqn.(3.8), $P(t)$ is:

$$\frac{dP(x)}{dt} = 2\lambda P(t) - qb^2P^2(t) + p^2 \tag{3.26}$$

$$P(T) = 0 \tag{3.27}$$

where, Eqn.(3.27) satisfies a Riccati type differential equation.

Therefore, from Eqs.(3.22) and (3.25), $v_f(t)$ is:

$$v_f(t) = \frac{-bK(t)C(t) + r_2\varphi(L_1)v_b}{r_1\varphi(0)} \tag{3.28}$$

As a results, Eqn.(3.28) represents a optimal forward speed of drive roll.

From the above description, a control function is derived as a standard speed function and linear combination, which is a function multiplied by a feedback gain to a tension value obtained from tension pick-up signals. Therefore, it is a high-feasibility system, and an actual control system is designed as per our proposed system.

4 Numerical example

Here, we present numerical examples of system dynamics and responses representing an optimal feedback gain. We again describe a system model, an evaluation function, and a Riccati-type differential equation.

- System dynamics

$$\frac{dx}{dt} = ax + bu$$
$$J = \int (px^2 + qu^2)dt$$

  where $u = -Ku$ and $K = q^{-1}bP$.

- Solution of Riccati type differential equation

$$\frac{dp}{dt} = 2aP - qb^2P^2 + p^2$$
$$\alpha_{1,2} = aB^2 \pm B\sqrt{a^2B^2 + p^2}$$
$$B = \sqrt{(ab^2)^{-1}}$$
$$P(t) = \frac{(\alpha_1 + K\alpha_2 e^{(\alpha_2-\alpha_1)t}/B^2)}{(1 + Ke^{(\alpha_1-\alpha_2)t}/B^2)}$$
$$K = -\left(\frac{\alpha_1}{\alpha_2}\right)$$

An optimal feedback function is calculated by Fig. 17, which shows a steady value of 0.0027. Fig. 18 shows the response of the optimal feedback system. The evaluation function value is approximately 7.189.

Table. 4: System parameters

a	b	P	q
1.5	0.1	0.3	1.0

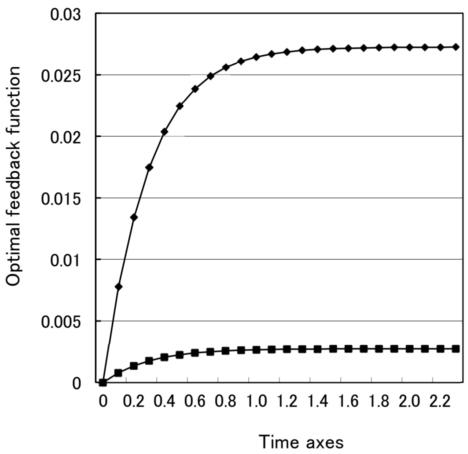

Fig. 17: Optima feedback function value

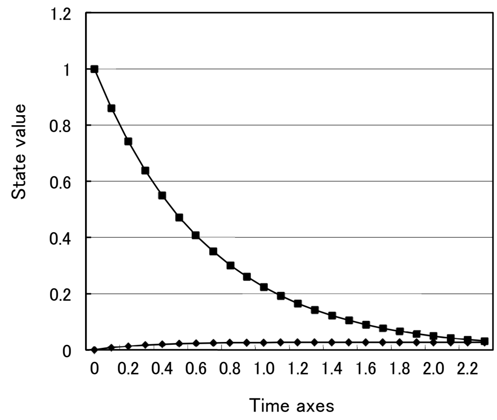

Fig. 18: Optimal feedback response value

5 Results

By treating the roll transport system as a distributed parameter system, we were able to clarify the tension point origin (or drive roll point). We proposed a mathematical model of the system and a highly feasible tension control system design. The mathematical model considered tension occurring at stands between rolls, and the tension control system is a distributed parameter model described by a PDE. The PDE is derived from certain equilibrium conditions that balance spatial forward deviation at stands between rolls and temporal variation of tension.

The main problem occurs with slipping in the driving roll that is attached to the sheet transport system, creating slackening (back tension) or heightening of tension (tension). Tension was quantified based on Young's modulus acting as a macro parameter and on the speed deviation between separate rolls.

In designing an optimal control system, we also considered a zero equilibrium regulator problem that takes into account the boundary control function. We utilized an optimal regulator theory derived by converting a PDE to an ordinary differential equation in a functional space. Consequently, it was possible to design a highly realistic system. However, a tension model for the peripheral speed difference of rolls between the drive roll and the variable is required. We are scheduled to report on this model in the next Bulletin.

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