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# MATHEMATICAL MODEL OF THERMAL REACTION PROCESS FOR EXTERNAL HEATING EQUIPMENT IN THE MANUFACTURE OF SEMICONDUCTORS (PART I)

Kenji Shirai[1], Yoshinori Amano[2] and Sigeru Omatu[3]

[1]Faculty of Information Culture
Niigata University of International and Information Studies
3-1-1, Mizukino, Nishi-ku, Niigata 950-2292, Japan
shirai@nuis.ac.jp

[2]Kyohnan Elecs Co., LTD.
8-48-2, Fukakusanishiura-cho, Fushimi-ku, Kyoto 612-0029, Japan
y_amano@kyohnan-elecs.co.jp

[3]Faculty of Engineering
Osaka Institute of Technology
5-16-1, Omiya, Asahi-ku, Osaka 535-8585, Japan
omatu@rsh.oit.ac.jp

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Abstract. *This paper presents a mathematical model for a thermal reaction process of external heating equipment. The new control system design for this process, which treats a heat source flowing model for an externally attached device is proposed. The equation of a distributed parameter system as a coupled system with the heat reaction process is presented. This paper proposes a lumped heating system to replace the distributed heating function with a time delay as a whole element. The model that is equivalent to a system in order to make possible practical devices is considered. This paper discusses the configuration of the linear model using the exact linearization method for the coupled system with external heating equipment.*
**Keywords:** Semiconductor manufacturing equipment, Thermal reaction process, External heating equipment, Distributed parameter system

1. **Introduction.** The Radio Corporation of America (RCA) has developed a metal-oxide-semiconductor (MOS) transistor and integrated MOS arrays, a silicon vidicon, storage tube, etc. In 1961, the RCA developed the removal of trace impurities on silicon lead to harmful effects electrically for the first time [1]. To develop an efficient framework of micro-device manufacturing for next-generation industries, it is necessary to have high-performance feedback control [2].

In particular, statistical process control (SPC) is a quality control method, which uses statistical methods, and is applied to monitor and control a process and to make a chart indicating the quality of the process. The monitoring and control of the process ensures that it operates at its full potential, and SPC is widely used in the manufacturing of semiconductors.

In the semiconductor manufacturing industry, an increasing number of suppliers have embarked on the development of large devices. In addition, there is a trend to produce large wafers, so the steps involved in each process are simultaneously performed on large chips. It is commonly known that the semiconductor manufacturing process is composed of a number of processes, and the thermal reaction process is one of the so-called front-end processes. Through this process, the semiconductor manufacturing equipment maintains a fixed temperature using a reaction liquid in wafer, and then, the desired efficiency is obtained.

In contrast, manufacturers have considered the application of a controller design method using output feedback in the presence of the bounded symmetric control, which is based on the linear matrix inequality (LMI).

In the chemical industry, petrochemical plants are involved in the manufacture of hydrogen from hydrocarbons [3]. In addition, the simultaneous optimization of scheduling and transportation is desired because of competition in the semiconductor industry. Therefore, it is important to reduce the production time and transportation cost [4].

This paper focuses to design a control system and a mathematical model for attached external heating equipment (AEHE) in the semiconductor manufacturing industry. In general, the heat reaction process is often used as a method of AEHE. An example of the system report of the thermal reaction process involves the combination of the thermal reaction model and the mass balance model [5]. In this example, the thermal reaction model is obtained by adding a new term to account for the exothermic reaction method by mixing various kinds of processing liquid. Exothermic reactions present unique hazards that occur within enclosed semiconductor equipment. The heat that is generated will increase the temperature of the reactants and products of the reaction, and that of surrounding materials. Because all substances have properties such as pressure that are temperature dependent, the resulting higher temperatures may affect the behavior of the materials in the environment.

In previous study, there are other systems such as a time-delay element that are attached to heating equipment [6].

In [7], it was reported that the multi-period repetitive control system is a type of servomechanism for periodic reference inputs. They proposed a design method for simple multi-period repetitive controllers for time-delay plants.

Therefore, this paper focuses on the mathematical modeling of the thermal reaction process, and considers a design structure for its control system. That is, this paper deals with the heat source flowing model as a model of AEHE. Further, we express it as an exact partial differential equation (PDE) because PDEs allow us to employ coupled systems for the thermal reaction process.

In order to make the model of the exothermic reaction system, the mathematical model of the thermal process as a coupled system with AEHE and the thermal reaction process are used.

The originality of this study is expressed for AEHE as a model of lumped parameter system, not a function of the spatial heating distributed system. This paper proposes a lumped heating system to replace the distributed heating function with a time delay as a whole element. It becomes an equivalent system in the construction of practical equipment.

Therefore, this paper discusses the configuration of the linear model using the exact linearization method for the coupled system with AEHE.

## 2. Mathematical Model of AEHE.

This section discusses the mathematical model of AEHE. Figure 1 shows a schematic of the external heating reaction system. This is with reference to the flow model reaction system. In Figure 1, $\xi \in [0, L]$ indicates space variable, $t \in [0, T]$ indicates time variable and $\Theta_c(t)$ indicates the internal temperature of the heated fluid in the heating unit. $\Theta_c(t)$ indicates the ambient temperature from AEHE. Further, Figure 2 shows the the cross-section view of Figure 1.

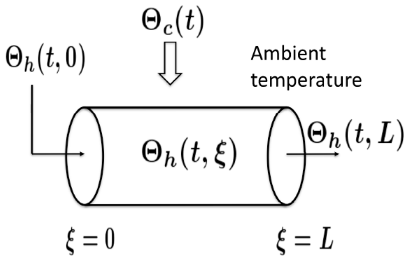


FIGURE 1. Schematic of AEHE

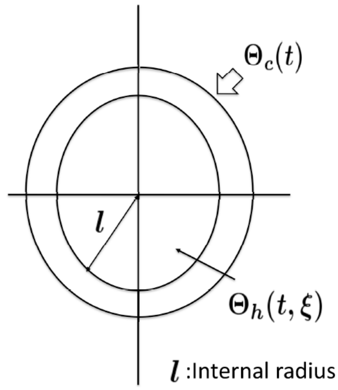


FIGURE 2. Cross-sectional view of TRP

Let the radiation coefficient of AEHE be $1/R$ and the specific heat of the fluid inside $C_M$. It is obtained by

$$\Delta\Theta_h(t,\xi) = \Theta_h(t+\Delta t,\xi) - \Theta_h(t,\xi) = \frac{U(\bullet)}{RC_M}\left\{\Theta_h(t,\xi) - \oint_{S_l}\Theta_c(t,\xi)d\xi\right\}\Delta t. \quad (1)$$

From Equation (1), it is obtained by

$$\frac{\Delta\Theta_h(t,\xi)}{\Delta t} = -\frac{U(\bullet)}{RC_M}\Theta_h(t,\xi) + \frac{2\pi l U(\bullet)}{RC_M}\Theta_c(t). \quad (2)$$

At this time, as Equation (2) describes the dynamic change in the fluid to move, the Lagrange differential equation is used, and it is obtained by

$$\frac{\partial\Theta_h}{\partial t} = \frac{\partial\Theta_h}{\partial t} + q\frac{\partial\Theta_h}{\partial\xi}. \quad (3)$$

Let rewrite Equation (2) as the following, and it is obtained by

$$\frac{\partial\Theta_h(t,\xi)}{\partial t} + q\frac{\partial\Theta_h}{\partial\xi} = \frac{U(\bullet)}{RC_M}\{2\pi l\Theta_c(t) - \Theta_h(t,\xi)\} \quad (4)$$

where, $q$ indicates advection velocity of the fluid.

Here, $U(t) \equiv U(\bullet)$ is called the overall heat transfer coefficient, which is a measure of the overall ability of a series of conductive and convective barriers to transfer heat [8]. It is commonly applied to the calculation of heat transfer in heat exchangers.

From this, let Equation (4) describe as

$$\frac{\partial\Theta_h(t,\xi)}{\partial t} + q\frac{\partial\Theta_h}{\partial\xi} = \frac{U(t)}{RC_M}\{k_0\Theta_c(t) - k_1\Theta_h(t,\xi)\} \quad (5)$$

where, let $k_0 > 0$, $k_1 > 0$, they indicate a reaction coefficient. Then, the initial condition of Equation (5) is

$$\Theta_h(0,\xi) = \Theta_h(\xi) \quad (6)$$

In addition, the boundary conditions of Equation (5) are

$$\Theta_h(t,0) = \Theta_{h_0}(t) \quad (7)$$

$$\Theta_h(t,L) = \Theta_{h_L}(t) \quad (8)$$

It is called AEHE of described above as a heating device of In-Line type. The unit of heating process thermal reaction is widely used in principle; however, its shape is different. Then, Figure 1 changes into Figure 3.

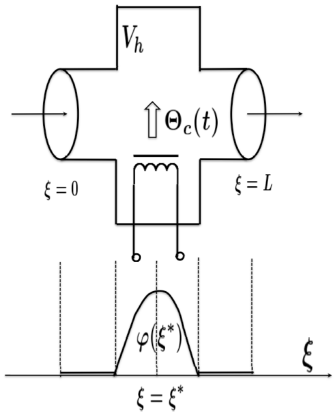


Figure 3. Centralized heating system

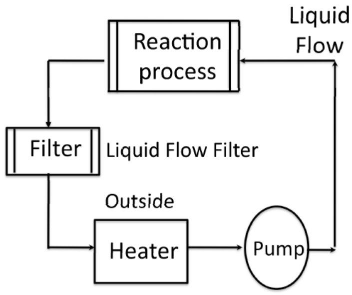


Figure 4. Schematic of reaction process

In other words, Figure 3 shows that the AEHE heats materials by the lumped state, not spatially state. Its distributed function indicates $\varphi(\xi^*)$, which is the function of $\varphi(\xi^*) \in C^1$. The shape of its function is described below. From this, Equation (5) is rewritten by

$$\frac{\partial \Theta_h(t,\xi)}{\partial t} + q\frac{\partial \Theta_h}{\partial \xi} = \frac{\varphi(\xi^*)}{RC_M} U(t)\{k_0\Theta_c(t) - k_1\Theta_h(t,\xi)\} \tag{9}$$

As described above, it was possible to express (9) as a conceptual model of AEHE. We will apply a mathematical model of heat reaction process using above model in the next chapter.

3. **Heat Reaction Process.** AEHE is described in this chapter, Figure 4 indicates its conceptual diagram.

The parameters used in this study define as follows.

**Definition 3.1.** *Definitions for various parameters*
$v_b = V_b + L_b S$: *Substantial volume of the reaction process unit* $[m^3]$
$v_h = V_h + L_h S$: *Substantial volume of the heater unit* $[m^3]$
$V_b$: *Substantial volume of the heater unit* $[m^3]$
$V_h$: *Substantial volume of the heater unit* $[m^3]$
$L_b$: *Effective length of the reaction process side* $[m]$
$L_h$: *Effective length of the heater unit* $[m]$
$S$: *Cross-sectional area of the pipe* $[m^2]$
$\Theta_b$: *Internal temperature of the reaction process* $[K]$
$\Theta_h$: *Internal temperature of the heating unit* $[K]$
$q$: *Flow rate* $[m^3 \cdot s^{-1}]$
$\rho$: *Density of the reaction mixture* $[kg \cdot m^{-3}]$
$c$: *Heat capacity of the reaction mixture* $[Cal \cdot Kg^{-1} \cdot K]$
$P$: *Input power of the reaction process* $[W]$
$H$: *Input power of the heating unit* $[W]$
$\Theta_0$: *Ambient temperature of the reaction process* $[K]$
$P_b$: *Exothermic reaction of the reaction process* $[W]$
$\kappa_0$: *Coefficient of heat dissipation to the outside* $[s^{-1}]$
$1/R$: *External radiation coefficient of the reaction mixture* $[s^{-1}]$
$C_M$: *Heat capacity of the heating unit* $[Cal \cdot Kg^{-1} \cdot K]$
$\Theta_c$: *Ambient temperature from the outside* $[K]$

*$U(t)$: Overall heat transfer function (control variable)*

The amount of heat that must be added during a chemical reaction in order to keep the desired temperature is considered as follows.

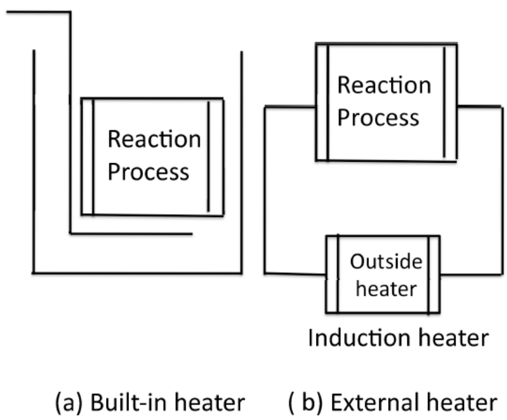


FIGURE 5. (a) Buit-in heater, (b) outside heater

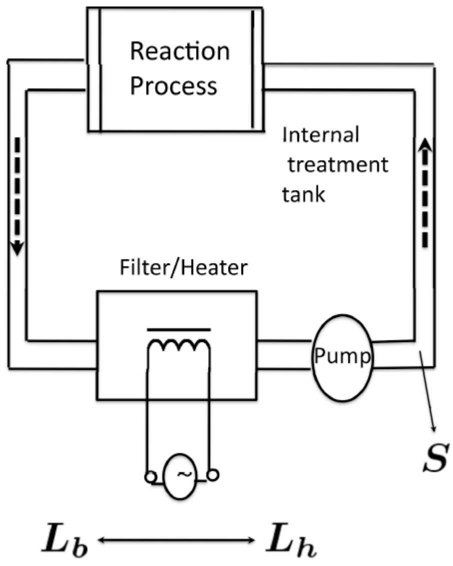


FIGURE 6. Detailed model of AEHE

The heat unit is attached the built-In type directly into the reaction process in Figure 5(a), and a thermal model can be expressed as

$$v_b \rho c \frac{d\Theta_b}{dt} = \kappa_0 v_b \rho c(\Theta_0 - \Theta_b) + P_b + P \tag{10}$$

where, let $\Theta_b(0) = \Theta_{b_0}$.

Such systems that are widely used conventional methods were used until a few years ago. However, their usage is again increasing.

In contrast, it is capable of heating the reaction liquid indirectly, instead of requiring that the heat exchange unit be installed directly into the reaction process in Figure 5(b).

Then, the above system model as Figure 5(b) can be shown as Figure 6.

In this case, the model that includes from the reaction process to AEHE is expressed by

$$v_b \rho c \frac{d\Theta_b}{dt} = \{-\Theta_b + \Theta_h(t, L))\} q\rho c + (\Theta_0 - \Theta_c)\kappa_0 v_0 \rho c + P_b \tag{11}$$

$$\frac{\partial \Theta_h(t,\xi)}{\partial t} + q(t)\frac{\partial \Theta_h(t,\xi)}{\partial \xi} = \frac{U(t)}{RC_M}\{k_0\Theta_c(t) - k_1\Theta_h(t.\xi)\} \tag{12}$$

where, let $U(t)$ be a overall heat exchange coefficient, let $\xi$ be a spatial variable at this time, and let $0 \leq \xi \leq L$, $L \cong L_b + L_h$. In this case, since AEHE is a heat exchanger in general, the derivation of the model is a strictly expressed for the system, then, it is derived by

$$\begin{aligned} \frac{\partial \Theta_h(t,\xi)}{\partial t} + q(t)\frac{\partial \Theta_h(t,\xi)}{\partial \xi} &= U(t)\left\{\frac{1}{RC_M}k_0\Theta_c(t) - \frac{k_1}{(V_h + L_h S)\rho_c}\Theta_h(t,\xi)\right\} \\ &\quad + \frac{1}{RC_M}\{k_0\Theta_c(t) - A\Theta_h(t,\xi)\} \end{aligned} \tag{13}$$

where, let $A$ be a effective area of heat exchange ($[m^2]$), $v_b = V_h + L_h S$, and $q(t)$ is the advection velocity of the fluid.

Then, let $\hat{U}(t) \equiv \varphi(\xi^*)U(t)$, $\varphi(\xi^*)$ is a distributed function in Equation (9). According to Equation (13), the outlet side mathematical model can be rewritten by

$$\frac{d\Theta_h(t,L)}{dt} = \frac{\varphi(\xi^*)U(t)}{RC_M}\{k_0\Theta_c(t) - k_1\Theta_h(t,L)\} + \frac{1}{RC_M}\{\Theta_0(t) - A\Theta_h(t,L)\} \tag{14}$$

where, let $RC_M = (V_h + L_h S)\rho_c$ [19].

Then, Equation (14) is rewritten by

$$\begin{aligned}\frac{d\Theta_h(t,L)}{dt} &= w(t,L)\{k_0\Theta_c(t) - k_1\Theta_h(t,L)\} + f\{\Theta_h(t,L)\}\\ &= w(t,L)g\{\Theta_h(t,L)\} + f\{\Theta_h(t,L)\}\end{aligned} \tag{15}$$

where, the following equation holds.

$$w(t,L) = \frac{\varphi(\xi^*)U(t)}{RC_M} \equiv w(t) \tag{16}$$

$$g\{\Theta_h(t,L)\} = k_0\Theta_c(t) - k_1\Theta_h(t,L) \tag{17}$$

$$f\{\Theta_h(t,L)\} = \frac{1}{RC_M}\{\Theta_0(t) - A\Theta_h(t,L)\} \tag{18}$$

In this case, by using the strict linearization method, Equation (15) can be rewritten by

$$\frac{dm(t)}{dt} = -am(t) + bu(t) \tag{19}$$

where, the following equation holds (see Appendix 1).

$$a = \left\{\frac{1}{RC_M} + \frac{k_1}{RC_M^2}\right\} \tag{20}$$

$$b = \frac{1}{RC_M} \tag{21}$$

$$w(t) = \frac{1}{\Theta_0(t) - k_1\Theta_h(t,L)} \times \left[\frac{1}{RC_M}\{\Theta_0(t) - k_1\Theta_h(t,L)\} + u(t)\right] \times \frac{A}{RC_M} \tag{22}$$

$$m(t) = k_1\Theta_h(t,L) - k_0\Theta_c(t) \tag{23}$$

Therefore, the model of the coupled system on Equations (11), (19) is

$$\begin{cases}\dfrac{dm(t)}{dt} = -am(t) + bu(t)\\ v_b\rho c\dfrac{d\Theta_b}{dt} = \{-\Theta_b + \Theta_h(t,L))\}q\rho c + (\Theta_0 - \Theta_c)\kappa_0 v_0\rho c + P_b\end{cases} \tag{24}$$

Thus, for example, if a recursive least squares estimation method is used, the parameters $a$ and $b$ in Equation (19) can be estimated. Moreover, the zero-order hold (ZOH) is a mathematical model of the practical signal reconstruction done by a conventional digital-to-analog converter (DAC). Equation (19) can be rewritten from the continuous data to the digital data by using ZOH. As a result, it is possible to configure the control system by using STC method [11].

Furthermore, to study the structure of the control system, Equation (24) can be rewritten as the following equation. The output variable sets $\hat{m}(t)$ when $\xi(t)$ is controlled by the

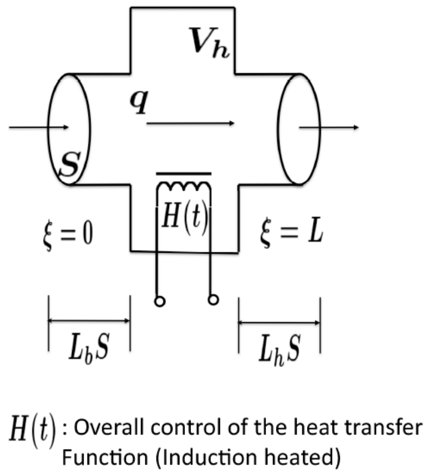


FIGURE 7. Conceptual model of the external heating

control variable and the mathematical model of the thermal reaction process is derived by

$$v_b \rho c \frac{d\Theta_b}{dt} = (-q\rho c + \kappa_0 v_0 \rho c)\Theta_b + \{(\hat{m}(t) + \Theta_c) q\rho c + \Theta_0 \kappa_0 v_0 \rho c + P_b\} \tag{25}$$

where, let $K_a = (q - \kappa_0 \cdot v_b)\rho c$.

Then, we obtain

$$\frac{d\Theta_b}{dt} = -\frac{K_a}{v_b \rho c}\Theta_b + \frac{1}{v_b \rho c}\{(\hat{m}(t) + \Theta_c) q\rho c + \Theta_0 \kappa_0 v_b \rho c + P_b\} \tag{26}$$

In addition, the variables in Equation (26) are translated as follows:

$$\begin{cases} \tilde{a} = \dfrac{K_a}{v_b \rho c}, \quad \tilde{b} = \dfrac{1}{v_b \rho c} \\ f(t) = \{(\hat{m}(t) + \Theta_c) q\rho c + \Theta_0 \kappa_0 v_b \rho c + P_b\} \end{cases} \tag{27}$$

From Equation (27), Equation (26) can be rewritten by

$$\frac{d\Theta_b}{dt} = -\tilde{a}\Theta_b + \tilde{b} f(t) \tag{28}$$

As described above, by specifying the system parameters in Equation (26) respectively, it is easy to examine the effects of substituting computer simulations of temperature of the reaction process in place of real equipment.

4. **Construction Method of Control System by the Strict Linear Method.** Here, we discuss the exact linear model.

Now, we rewrite Equation (28) to

$$\frac{d\Theta_b}{dt} + \tilde{a}\Theta_b = \tilde{b} f(t) \tag{29}$$

In this case, Equation (29) can be rewritten from continuous data to digital data by using ZOH. It is obtained by

$$(1 - \exp(-\tilde{a}T) \cdot z^{-1})\Theta_b = \frac{\tilde{b}}{\tilde{a}}(1 - \exp(-\tilde{a}T)) \cdot z^{-1} \cdot f(t) \tag{30}$$

where, let $z^{-1} = \exp(-sT)$. Then, let $\alpha = -\exp(-\tilde{a}T)$, $\beta = \frac{\tilde{b}}{\tilde{a}}(1-\exp(-\tilde{a}T))$. Therefore, Equation (30) is as

$$(1+\alpha)\Theta_b = \beta \cdot z^{-1} \cdot f(t) \tag{31}$$

That is, it is obtained by

$$\Theta_b(t) = -\alpha\Theta_b(t-1) + \beta \cdot f(t-1) \tag{32}$$

Here, the predicted value $\Theta_b^*(t+1|t)$ in the left-hand side of Equation (32) is derived by

$$\Theta_b^*(t+1|t) = -\alpha\Theta_b(t) + \beta \cdot f(t) \tag{33}$$

From Equation (32), the prediction error is derived by

$$\epsilon(t+1|t) = \Theta_b^*(t+1|t) - \gamma(t) = -\alpha\Theta_b(t) + \beta \cdot f(t) - \gamma(t) \tag{34}$$

Therefore, to minimize Equation (34), $f(t)$ that minimize the mean square prediction error is derived by

$$f(t) = \frac{\alpha\Theta_b(t) + \gamma(t)}{\beta} \tag{35}$$

where, $\gamma(t)$ is a target desired value for the optimal system. In this case, from Equations (27) and (35), it is obtained by

$$\frac{\gamma(t) - \exp(-\tilde{a}T)\Theta_b(t)}{\frac{\tilde{b}}{\tilde{a}}(1-\exp(-\tilde{a}T))} = (\hat{m}(t) + \Theta_c(t))q\rho c + \Theta_0\kappa_0 v_b \rho c + P_b \tag{36}$$

From this, according to track the $\gamma(t)$ by the $\Theta_b(t)$, let the target desired value of the $m(t)$ be the $\hat{m}(t)$. Then, from Equation (36), it can be obtained by

$$\hat{m}(t) = \frac{1}{q\rho c}\left\{\frac{\gamma(t) - \exp(-\tilde{a}T)\Theta_b(t)}{\frac{\tilde{b}}{\tilde{a}}(1-\exp(-\tilde{a}T))} - \Theta_0\kappa_0 v_b\rho c - P_b\right\} - \Theta_c \tag{37}$$

Applying Equation (19) in the same manner analysis and according to Equation (35), it can be obtained by

$$u(t) = \frac{\hat{m}(t) - \exp(-aT)m(t)}{\frac{b}{a}(1-\exp(-aT))} \tag{38}$$

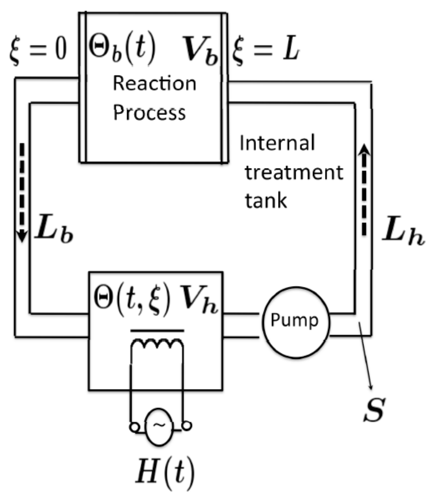


Figure 8. System model of the external heat system

Therefore, from Equations (16) and (22), it can be obtained by

$$\frac{\hat{U}(q,H,t)}{RC_M} = \frac{1}{[\Theta_0 - k_1\Theta_h(t,L)]} \times \left[\frac{1}{RC_M}\{k_0\Theta_c(t) - k_1\Theta_h(t,L)\} + u(t)\right] \times \frac{1}{RC_M} \quad (39)$$

If $q$ is stable variable in $\hat{U}(q,H,t)$, Equation (39) can be derived by approximately

$$\frac{\hat{U}(q,H,t)}{RC_M} = n(q)\varphi(\xi)H(t), \quad \forall n > 0 \quad (40)$$

where, $n(q)$ is the linear function that $q$ depends on $\hat{U}(t)$.

Above describe, using Equation (40) from Equation (37), it can be obtained from the control input $H(t)$ that inputs to AEHE. Here, the distribution function of AEHE is defined as follows:

**Definition 4.1.** *Distribution function of AEHE*

$$\varphi(\xi^*) = c_1 \exp\left\{-(\xi^* - K)^2\right\}, \forall|\xi^*| < K \quad (41)$$

*where, if* $|\xi^*| \geq K$, *let* $\varphi(\xi^*) = 0$.

Assuming that $0 \leq \xi \leq L \in \Omega$, let $\varphi \in \mathcal{D}(\Omega)$, $\varphi$ in Equation (41) is a real function with a compact support in $\Omega$.

Because Equation (40) is a smooth function $\varphi(\xi^*)$, such as Equation (41), the AEHE represents the mathematical control function that heats with the lumped state spatially.

According to this, by using Equations (38)-(41), the electric input power $H(t)$ can be obtained by

$$H(t) = \frac{[n(q)\varphi(\xi^*)]^{-1}}{[\Theta_0 - k_1\Theta_h(t,L)]} \times \left[\frac{1}{RC_M}\{k_0\Theta_c - k_1\Theta_h(t,L)\} + u(t)\right] \quad (42)$$

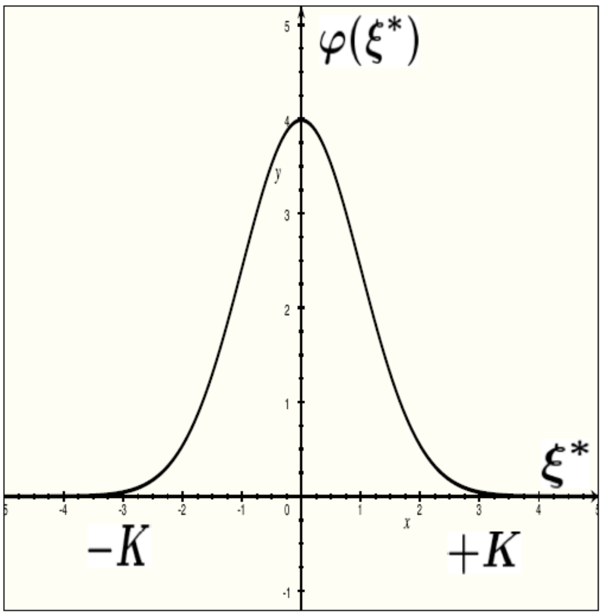


FIGURE 9. Distribution function of the heating

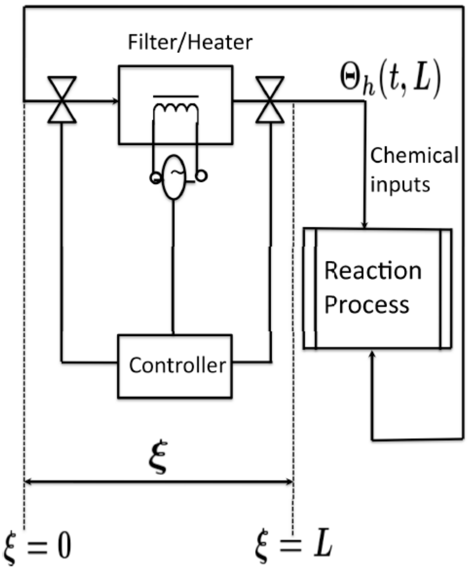


FIGURE 10. Reaction diagram of the process fluid supply

5. **Configuration of the Control System of Heat Flow.** From Equation (13), the variables of $\hat{U}$ is strictly derived by

$$\frac{\partial\Theta_h(t,\xi)}{\partial t} + q(t)\frac{\partial\Theta_h(t,\xi)}{\partial\xi} = \frac{1}{RC_M}U(t)\{k_0\Theta_c(t) - k_1\Theta_h(t,\xi)\} + \frac{U(t)}{RC_M}\{k_0\Theta_c(t) - A\Theta_h(t,\xi)\} \tag{43}$$

where, $U(t)$ can be derived by the following function:

$$U(t) \equiv U(q, I, V, f, t) \tag{44}$$

The temperature $\Theta_h(t,\xi)$ of the reaction liquid can be obtained from Equations (43) and (44). The variables $V(\bullet)$, $f$ in Equation (44) are the current and frequency in the inductive method respectively.

That is, by spraying the reaction liquid, the object temperature can be derived by $\Theta_h(t,L)$.

Here is the time-optimal heat control problem, and its accuracy of the control temperature is required.

As mentioned above, the object is heated over a long period of time in the circulatory system, and the temperature control is obtained by securing the more reaction liquid in the tank.

To reduce a die size, the framework of the semiconductor process has evolved and have been migrated to the thermal reaction process.

Now, before we discuss the control algorithm, the electrical variables for the induction heat unit are described. Figures 11-13 and the meaning of the variables used in the following equation is as follows. $V(\bullet)$: Supply voltage, $I(A)$: Circuit current, $P(W)$: Input power, $f$: Power frequency, $\delta$: Skin effect coefficient, $C$: Capacitance, $V_s$: Electrode area, $d$: Distance between electrodes, $\epsilon_s$: Dielectric constant of the object to be heated, $\epsilon_0$: Permittivity of vacuum.

First, the input power is given by

$$P = VI\cos\left(\frac{\pi}{2} - \delta\right) = VI\sin\delta \cong VI\tan\delta(Watt), \quad \forall\delta << 1 \tag{45}$$

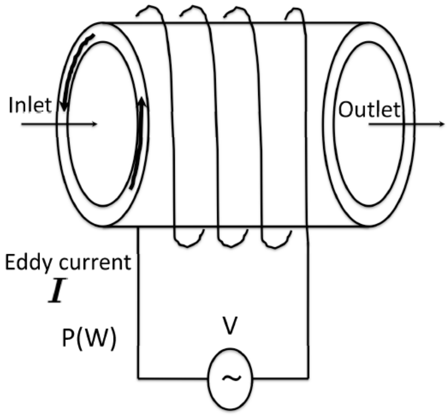


FIGURE 11. Schematic of the induction heating

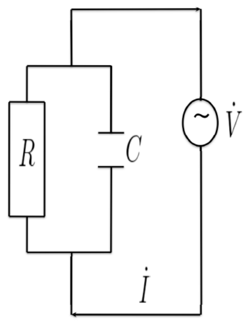


FIGURE 12. The equivalent circuit of the induction heating

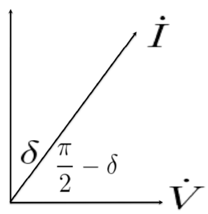


FIGURE 13. Phase difference $\delta$

In addition, $c = \epsilon_0 \epsilon_s \frac{s}{d}$, $I = \omega c V$; therefore, the following equation holds.

$$I = \omega c V = \frac{5}{9}\epsilon_s f s \frac{V}{d} \times 10^{-10}(A) = \frac{5}{9}\epsilon_s f s E \times 10^{-10}(A), \ \ \forall E = \frac{V}{d} \tag{46}$$

where, let $\epsilon_0 = 8.855 \times 10^{-12}[F/m] = 1/(4\pi \times 9 \times 10^9)$. In addition, $E$ represents a electrolytic strength. Therefore, power $P$ is derived by

$$P = \frac{5}{9} ds E \epsilon_s \tan\delta \times 10^{-10}(W) \tag{47}$$

where, $ds$ represents the Dielectric volume of the object to be heated. Therefore, the power per unit volume is derived by

$$P = \frac{5}{9} f E^2 \epsilon_s \tan\delta \times 10^{-10}(W) \tag{48}$$

where, $\epsilon_s \tan\delta$ represents the Dielectric loss factor. Therefore, power $P$ can also be expressed by

$$P \cong f E^2 \epsilon_s \tan\delta \tag{49}$$

where, $f, E, \epsilon_s \tan\delta$ is the variable determined by the object.

Therefore, from Equations (42)- (49), $u(t)$ can be obtained.

At this time, the system model that can be obtained by the Exact Linearization Method, and are given by Equation (19) and Equations (20)-(23). From above these equations, we obtain

$$\frac{dm(t)}{dt} + am(t) = bu(t) \tag{50}$$

where, let $m(t) = k_1\Theta_h(t, L) - k_0\Theta_c$, and let the $\Theta_c(t) \equiv const$. For the simplicity. The parameters $a, b$ are the fixed values in Equations (20) and (21) respectively. In addition, from Equation (16) and Equation (22), it is obtained by

$$\frac{U(I, f, q, t)}{RC_M} = \frac{1}{[\Theta_0 - k_1\Theta_h(t, L]}\left[\frac{1}{RC_M}(k_0\Theta_c - k_1\Theta_h(t, L)); u(t)\right] \times \frac{1}{RC_M} \tag{51}$$

In this way, it was found that the temperature of the object can be controlled by the overall heat transfer function $U(t)$. At this time, the control parameters can be done by the input voltage and frequency.

Then, by applying the maximum principle using the model described above, the flow control in the time-optimal control system is discussed.

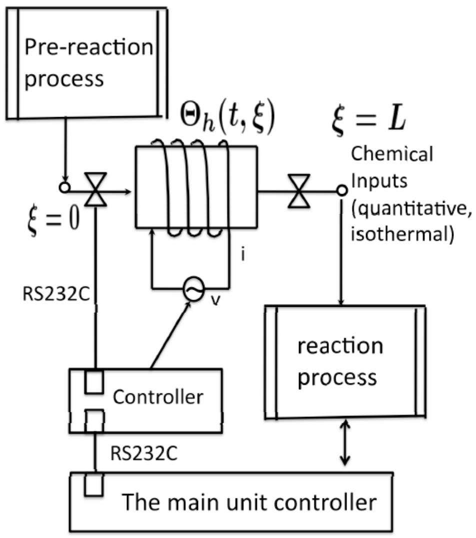


FIGURE 14. Reaction process control system equipment

5.1. **Solution by the maximum principle.** In this chapter, though it is different from the control method STC, the constraint condition of the following control function is discussed.

$$0 \le U(t) \le M \tag{52}$$

Regarding with a control criteria, the criteria is derived by

$$J = \min_{m(t)}\left[\frac{1}{2}\int_0^T (r - m(t))^2 dt\right] \tag{53}$$

where, $r$ is the desired value.

In this case, the Hamiltonian is derived by

$$H = \frac{1}{2}(r - m(t))^2 - \lambda(-am(t) + bu(t)) \tag{54}$$

$$\frac{d\lambda}{dt} = -\frac{\partial H}{\partial m} = rm(t) - \frac{1}{2}m(t)^2 + \lambda am(t) = \left(r - \frac{1}{2} + \lambda\right)m(t) \tag{55}$$

$$\frac{dm}{dt} = -\frac{\partial H}{\partial \lambda} = -am(t) + bu(t) \tag{56}$$

The optimal control function $u_{opt}(t)$ is also derived by

$$u_{opt}(t) = \frac{\partial H}{\partial u} = -b\lambda(t) \tag{57}$$

$$u_{opt} \equiv \begin{cases} 0 : \lambda(t) > 0 & (U(t) = 0) \\ M_u : \lambda(t) < 0 & (U(t) = M) \end{cases} \tag{58}$$

where, let $0 \leq u(t) \leq M_u$. At this time, $\lambda(T) = 0$ is a terminal condition.

6. **Conclusion.** As mentioned above, we proposed an exact mathematical thermal model for thermal reaction processes, such as those discussed in this study. In addition, a model that is in agreement with a real system was also proposed using the exact linearization method.

Prior to the validation of our method, simulations were carried out. In this way, by taking advantage of the optimal control theory or various mathematical programming techniques, it was possible to configure the control system.

The originality of this study was the expression of the mathematical model of AEHE and the expression of a coupled system for reaction processes.

## REFERENCES

[1] W. Kern, Cleaning solutions based on hydrogen peroxide for use in silicon semiconductor technology, *RCA Review*, vol.31, pp.187-206, 1970.

[2] S. Kurogi, A simplified competitive associative net and a model-switching predictive controller for temperature control of chemical solutions, *ICONIP 2000*, vol.2, pp.791-796, 2000.

[3] N. Kamil, A. Bouhafid and A. Benzaouia, Control of an hydrogen reformer with outout feedback: An LMI approach, *International Journal of Innovative Computing, Information and Control*, vol.4, no.4, pp.785-792, 2008.

[4] T. Furusho, T. Nishi and M. Konishi, Distributed optimization method for simultaneous production scheduling and transportation routing in semiconductor fabrication bays, *International Journal of Innovative Computing, Information and Control*, vol.4, no.3, pp.559-575, 2008.

[5] S. Kuroki, K. Nishida, H. Nobutomo, T. Sakamoto, M. Mitsumata and K. Ito, A thermal model of the RCA cleaning system and adaptive predictive temperature control of cleaning solutions, *Journal of SICE*, vol.37, no.8, pp.754-762, 2001.

[6] T. Kubo, Insensitivity of the optimal memoryless regulator of systems with time-delay in the states, *International Journal of Control*, vol.74, no.3, pp.205-210, 2001.

[7] K. Yamada, N. Li, M. Kobayashi and H. Takenaga, A design method for simple multi-period repetitive controllers for time-delay plants, *International Journal of Innovative Computing, Information and Control*, vol.5, no.10(B), pp.3313-3328, 2009.

[8] T. Aihara, *Heat Transfer Engineering*, SHOKABO PUBLISHING Co., Ltd., 1994.

[9] T. Tsuga, Advanced wet clean-up process for ULSI fabrication: Particle free single wafer cleaning system, *Technical Report of IEICE, SDM, Silicon Material Device*, vol.98, no.243, pp.69-75, 1998.

[10] R. N. Rogers, Differential scanning calorimetric determination of kinetics constants of systems that melt with decomposition, *Thermochimica Acta*, vol.3, pp.437-447, 1972.

[11] S. Omatu, K. Imai and T. Yamamoto, Temperature control by using STC, *The 29th Japan Joint Automatic Control Conference*, 1986.

[12] T. Ohmi, Total room temperature wet cleaning of Si surfaces, *Semiconductor International*, vol.8, pp.323-328, 1996.

[13] R. Matsumoto, Y. Sato, M. Kamo and N. Setaka, Vapor deposition of diamond particles from methane, *J. Appl. Phys*, vol.21, pp.183-185, 1982.

[14] M. Kamo, Y. Sato, S. Matsumoto and N. Setaka, Low-pressure deposition of diamond thin films, *J. Cryst. Growth*, vol.62, pp.642-644, 1983.
[15] B. V. Spitsyn, L. L. Bouelov and B. V. Derjaguin, Vapor growth of diamond on diamond and other surfaces, *J. Cryst. Growth*, vol.52, pp.219-226, 1981.
[16] H. Vora and T. J. Moravec, Structural investigation of thin films of diamondlike carbon, *J. Appl. Phys*, vol.52, pp.6151-6157, 1981.
[17] S. Aisenberg and R. Chabot, Ion-beam deposition of thin films of diamondlike carbon, *J. Appl. Phys*, vol.42, pp.2953-2958, 1971.
[18] M. Ohno, Examples of heat transfer equipment design, *Chemical Equipment*, vol.4, pp.31-38, 1984.
[19] Y. Amano, *Optimal Control of Distributed Parameter System*, Ph.D. Thesis, Ritsumeikan University, 1997.
[20] M. Shima, S. ikeda and M. Okamoto, Optimal control of bilinear distributed parameter system, *ISCIE*, vol.17, no.11, pp.689-697, 1973.
[21] Y. Sawaragi, H. Fukawa and S. Katuma, Optimization of one-dimensional nonlinear control system with control variables appearing linearly, *ISCIE*, vol.15, no.2, pp.148-154, 1970.
[22] B. F. Schutz, *Geometrical Methods of Mathematical Physics*, Cambridge University Press, 1980.

**Appendix 1: Derivation of the parameters a and b.** In Exact Linearization Method, we take advantage of Lie derivative.

In vector field, Lie brancket $[f, g](x)$ of vector-valued function $f(x)$ and $g(x)$ respectively are

$$[f, g](x) = \frac{\partial g}{\partial x} f(x) - \frac{\partial f}{\partial x} g(x) \tag{59}$$

where, Formula (59) is defined in vector field.

$ad_f^i g(x)$ is

$$ad_f^0 g(x) = g(x), \tag{60}$$
$$ad_f^1 g(x) = \left[f, ad_f^{i-1} g\right](x) \tag{61}$$

where, Formula (61) is also defined in vector field.

In addition, the Lie derivative of scalar function $\phi(x)$ in vector field $f(x)$ is

$$L_f \phi(x) = \frac{\partial \phi}{\partial x} f(x) \tag{62}$$

Then, by running the Lie derivative repeatedly, we obtain

$$L_f^1 \phi(x) = L_f \phi(x), \tag{63}$$
$$L_f^i \phi(x) = L_f \left[L_f^{i-1} \phi(x)\right]. \tag{64}$$

In such a model of this study, we put

$$g(\Theta(t, L)) = k_0 \Theta_0(t) - k_1 \Theta(t, L), \tag{65}$$
$$f(\Theta(t, L)) = \frac{1}{RC_M} (\Theta_c(t) - \Theta(t, L)) \tag{66}$$

Therefore, it is to satisfy the theorem of Exact Linearization Method [22].

Consequently, we obtain

$$\begin{aligned} Lad_f^0 g\phi(\Theta(t, L)) &= \frac{\partial \phi}{\partial \Theta} ad_f^0 g(\Theta(t, L)) \\ &= \frac{\partial \phi}{\partial \Theta} (k_0 \Theta_0(t) - k_1 \Theta(t, L)), \end{aligned} \tag{67}$$

$$
\begin{aligned}
Lad_f^1 g\phi(\Theta(t,L)) &= \frac{\partial \phi}{\partial \Theta} ad_f^1 g(\Theta(t,L)) \\
&= \frac{\partial \phi}{\partial \Theta} \times \frac{1}{RC_M}(k_0\Theta_0(t) - k_1\Theta_c)
\end{aligned} \tag{68}
$$

Therefore, we put

$$
\phi(\Theta(t,L)) = k_0\Theta_0 - k_1\Theta(t,L) \tag{69}
$$

Then, $w(t)$ becomes

$$
\begin{aligned}
w(t) &= \left[\frac{L_f^1\phi(\Theta(t,L))}{L_g L_f^0\phi(\Theta(t,L))} + \frac{u(t)}{L_g L_f^0\phi(\Theta(t,L))}\right] \times \frac{A}{RC_M} \\
&= \left[\frac{\{RC_M\}^{-1}(\Theta_c - \Theta(t,L))}{k_0\tau_{l_i}^c - k_1\Theta(t,L)} + \frac{u(t)}{k_0\Theta_c - k_1\Theta(t,L)}\right] \times \frac{A}{RC_M} \\
&= \frac{1}{k_0\Theta_0(t) - k_1\Theta(t,L)}\left\{\frac{1}{RC_M}(\Theta_c - \Theta(t,L)) + u(t)\right\} \times \frac{A}{RC_M}
\end{aligned} \tag{70}
$$

where, let $\hat{\Theta} = \phi(\Theta(t,L)) \equiv \Theta(t,L) - \Theta_c(t)$, and we can rewrite to

$$
\begin{aligned}
\frac{d\hat{\Theta}}{dt} &= \frac{d\hat{\Theta}}{d\Theta} \cdot \frac{d\Theta}{dt} \\
&= \frac{1}{RC_M}(\Theta_0(t) - k_1\Theta(t,L)) + (k_0\Theta_c(t) - k_1\Theta(t,L)) \\
&\quad \times \left[\frac{A/C_M}{k_0\Theta_0(t) - k_1\Theta(t,L)} \cdot \frac{1}{RC_M}\{\Theta_0(t) - \Theta(t,L)\} + u(t)\right] \\
&= -\left\{\frac{1}{RC_M} + \frac{A}{RC_M^2}\right\} \cdot \left\{\Theta_c - \Theta(t,L)\right\} + \frac{A}{RC_M}u(t) \\
&= -\left[\frac{1}{RC_M} + \frac{A}{RC_M^2}\right]\hat{\Theta}(t) + \frac{A}{RC_M}u(t) \\
&= -a\hat{\Theta}(t) + bu(t)
\end{aligned} \tag{71}
$$

where,

$$
a = \left[\frac{1}{RC_M} + \frac{A}{RC_M^2}\right], \quad b = \frac{A}{RC_M} \tag{72}
$$

$$
\begin{aligned}
w(t) &= \frac{1}{k_0\Theta_0(t) - k_1\Theta(t,L)} \\
&\quad \times \left\{\frac{1}{RC_M}\{\Theta_0(t) - k_1\Theta(t,L)\} + u(t)\right\} \times \frac{A}{RC_M}
\end{aligned} \tag{73}
$$

where, $w(t)$ become

$$
w(t) = \frac{A}{RC_M}\hat{W}(t) \tag{74}
$$

That is, the model of Exact Linearization Method is expressed by

$$
\frac{d\hat{\Theta}(t)}{dt} = -a\hat{\Theta}(t) + bu(t). \tag{75}
$$