International Journal of Innovative Computing, Information and Control
Volume **9**, Number **11**, November **2013**
ICIC International ©2013 ISSN 1349-4198
pp. 4431–4445

PROCESS THROUGHPUT ANALYSIS FOR MANUFACTURING PROCESS UNDER INCOMPLETE INFORMATION BASED ON PHYSICAL APPROACH

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Received February 2012; revised June 2012

Abstract. *In order to analyze a manufacturing process, the present paper makes discussion from a physical point of view, not from a conventional management engineering point of view. That is, an idea of a production level corresponding to an energy level being discussed in physics is introduced. When information needed over a whole manufacturing process period is not available 100, an expected value and variance of throughput of the whole process period is estimated by utilizing Kalman filter theory having been used for a state estimation problem in the control theory. Process throughput is evaluated by taking an estimated expected value and variance of a throughput rate as an evaluation criterion, and a cash flow proportional to a manufacturing period as an evaluated value. Finally, a result of numerical value simulation of process throughput evaluation is shown.*
Keywords: Process throughput, Energy level, Stochastic process, Ito's lemma, Kalman filter

1. **Introduction.** As physical research about productivity improvement in manufacturing processes in the manufacturing industry, there is a research related to modeling of cost (interest expense, depreciation allowance and the like) needed for fund raising and investment in a certain company [1]. The research has introduced a stochastic Euler-Lagrange equation in order to characterize both of dynamics of a total investment amount and impact of various political measures on capital accumulation based on data of a Japanese gas company in the period from 1981 to 1995. In addition, it has proposed a dynamic factor demand model in order to analyze a dynamic cost structure. It can be said that it is a very interesting research.

Also, as research related to financial analysis, there is a report in which, in order to compare a rate of return and variance of short term investment, a rate of return and variance at the time of long term investment was researched [2]. In this research, Monte Carlo method was utilized in order to simulate a rate of return. Further, there is a report saying that, as a result of investigation of long-term return on investment and its dispersion characteristics, geometric Brownian motion models describing a price of risk assets differ substantially from actual phenomenon [3]. Also in this research, Monte Carlo method was utilized in order to simulate a rate of return. In any of such research, research about analysis of a rate of return or a dynamic cost structure was performed based on investment.

In addition, because there had been no research about stochastic volatility estimation of Heston model, Aihara, Bagchi and Saha carried out volatility estimation of Heston model, and presented a realization method of that. Its target was stock prices, not Kalman Filter.

There are a lot of researches about state estimation using Kalman Filter in the control theory field. The research by Takeuchi and Hirata is not research in the financial field [5].

In [6], a control problem of a counting process having a jump process is handled using Ito-type linear stochastic differential equation. In the past, Ito-type linear stochastic differential equations were also a basis of a pricing theory of financial theories.

The motive of the present research that has led to promote such research during many years of experience of general industrial machine control equipment manufacturing business is as follows. There was no physical discussion about production processes that had been accumulated through equipment manufacturing business. We reported a way of thinking that, in a process manufacturing a product ordered from an orderer in the manufacturing industry, it is constituted of product elements for which production is assorted among processes. In other words, when move of a product element having been manufactured in one process to the next process is called a production flow, we reported that, by thinking that this production flow is displacement of production density in a unit production direction, an equation dominating a manufacturing process is indicated by a diffusion equation [7].

In the present paper, a given control equipment is ordered from a customer, then manufactured in a manner classified into a number of production elements, and a finished product is delivered to the customer. The feature of the present paper is in a point that production elements in manufacturing processes are treated as stochastic production operation. In particular, in order to analyze a manufacturing process as a stochastic process, we have introduced an idea of a production level corresponding to an energy level being discussed in physics. A valence electron transits to a conducting state due to a rise in potential (transition of a manufacturing process), and lowers an energy level by radiating energy with time. On this occasion, radiated energy is made to correspond to a phenomenon to produce business return. When the Fermi level of a valence electron is high, a conduction electron density is increased, and a positive hole density reduces. Similarly, if operations from the order entry of a product to completion of the manufacturing processes proceed without delay, high return can be obtained. Increase of conduction electrons corresponds to increase of production density, and decrease of positive hole density corresponds to increase of return. As state transition of a valence electron is being analyzed stochastically, it is often also in the manufacturing industry that order entry delays, and deviation arises in planned manufacturing operations. Occurrence of such unexpected event will be handled as a stochastic event.

In a company, it is important to determine a rational throughput rate for continuation of production under an incomplete information state. In the present research, aiming at rationally performing start date management in the manufacturing industry, a mathematical model of throughput is formulated based on data, and a mathematical structure of start date management is made clear to some extent.

According to this result, it is shown that Kalman filter theory having been used in a state estimation problem in the control theory conventionally can be applied under an incomplete information state. In addition, by applying a theory of ongoing assessment in Real Option, a determination condition of a throughput rate is made clear and is confirmed by numerical value calculation.

As above, it is considered to be a great outcome that the achievements obtained in the state estimation problem in the control theory were applied to production processes, and a cash flow was able to be evaluated based on an estimated throughput rate.

2. **Production Framework in Equipment Manufacturer.** We refer to the production system in manufacturing equipment industry studied in this paper. This is not a special system but "Make-to-order system with version control". Make-to-order system is a system which allows necessary manufacturing after taking orders from clients, resulting in "volatility" according to its delivery date and lead time. In addition, "volatility" occur in lead time depending on the contents of make-to-order products (production equipment).

However, effective utilization of the production forecast information on the orders may suppress certain amount of "variation", but the complete suppression of variation will be difficult. In other words, "volatility" in monthly cash flow occurs and of course influences a rate of return in these companies. Production management systems, suitable for the separate make-to-order system which is managed by numbers assigned to each product upon order, is called as "product number management system" and is widely used.

All productions are controlled with numbered products and instructions are given for each numbered products.

Thus, ordering design, logistics and suppliers are conducted for each manufacturer's serial numbers in most cases except for semifinished products (unit incorporated into the final product) and strategic stocks.

Therefore, careful management of the lead time or production date may not suppress "volatility" in manufacturing (production).

The company in this study is the "supplier" in Figure 1 and "factory" here. Companies are under the assumption that there are N (numbers of) suppliers; however, this study deals with one company because no data is published for the rest of the companies $(N-1)$.

In Figure 1, given that a deal asking price from a customer is $\Phi(s_i)$ and supplier's internal cost is $C_i(s_i)$, a rate of return is indicated.

$$h(s_i) \equiv \frac{\Phi(s_i) - C_i(s_i)}{\Phi(s_i)}. \tag{1}$$

Therefore, the average of $\{h(s_i), i = 1, 2, \cdots, N\}$ is

$$\hat{h}(s_i) \equiv \frac{1}{N}\sum_{i=1}^{N} h(s_i). \tag{2}$$

3. **Production Energy Level and Production Density Function.** Here, by applying an energy level discussed in physics to production business, a production level is cited. Figure 2 indicates a transition chart of a production energy level. E_P is a work trend level, E_T is a work pursuance determination hazard level, that is, a critical value for judgment as to whether work is possible or not, and there is a case where work is conducted at this level. E_M is a work start level, and E_L indicates a level unworthy of starting work. E_L or less indicates a stop level. These definitions correspond to evaluation reference values in the industrial engineering field.

Here, let $W_i \equiv h(s_i)$, following formula is

$$W \equiv \{W_1, W_2, \cdots, W_N\} \in R^N. \tag{3}$$

In Figure 3, a production density function shows normal distribution such as

$$y = \frac{10}{\sqrt{2\pi}\sigma}\exp\Big(-\frac{(x-3)^2}{2\sigma^2}\Big), \quad \sigma = 1. \tag{4}$$

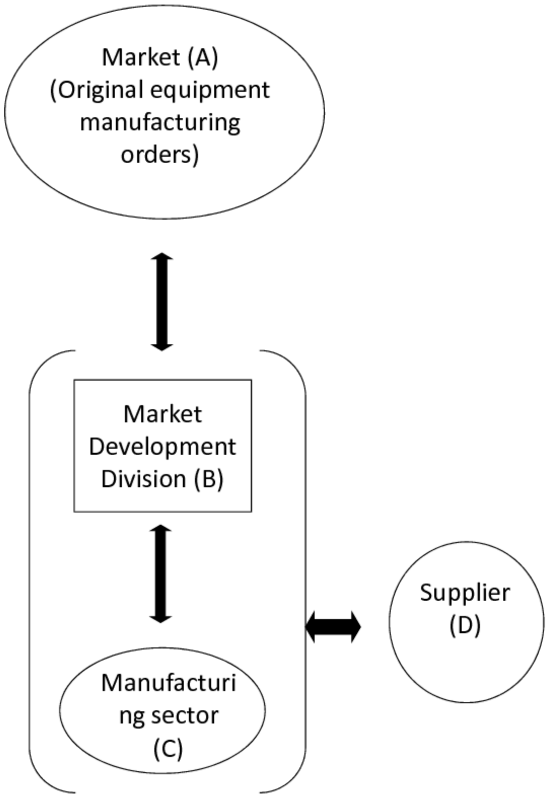

FIGURE 1. Business association chart of company of re-search target

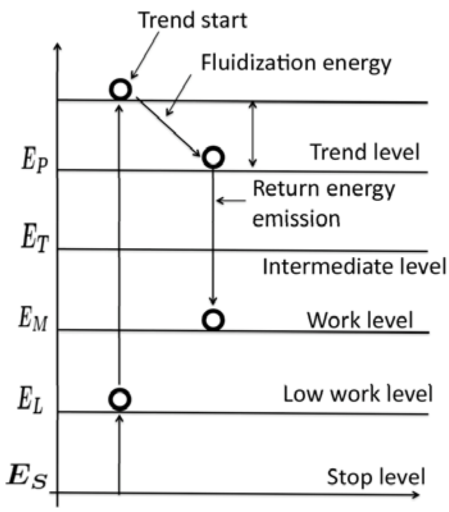

FIGURE 2. Production energy level transition chart

In Figure 3, a rate of return density function shows log-normal distribution such as

$$y = \frac{10}{\sqrt{2\pi}\sigma} \exp\Big(-\frac{\ln(x-3)^2}{2\sigma^2}\Big), \quad \sigma = 1. \tag{5}$$

Here, a production density function that exists on the work trend level E_P is defined as

Definition 3.1. *Production density function*

$$S_{rp}(E_P) = N_{SP}(E_P) \cdot F_{SP}(E_P), \quad \forall E = E_P. \tag{6}$$

4. **Stochastic Analysis of Production Process.** A production density function is generally given by

$$S_{rp}(E) = N_{SP}(E) \cdot F_{SP}(E) \tag{7}$$

where, N_{SP} indicates a production density constituting a single product. $F_{SP}(E)$ indicates a probability distribution function of a production density, and this production density is the probability distribution function existing on the reference value EP (refer to Figure 4).

Therefore, the number of states existing in $[E_P, E_P + \Delta E_P]$ can be expressed by

$$Z_s(E) = \frac{\partial N_{SP}(E)}{\partial E}. \tag{8}$$

In addition, a production density function existing on E_P can be calculated as

$$S_{rp} = \int_{E_P}^{\infty} N_{SP}(E) \cdot F_{SP}(E) dE. \tag{9}$$

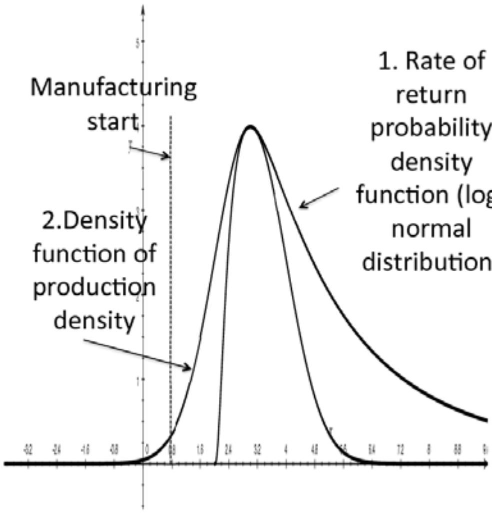

FIGURE 3. Production density function and rate of return density function

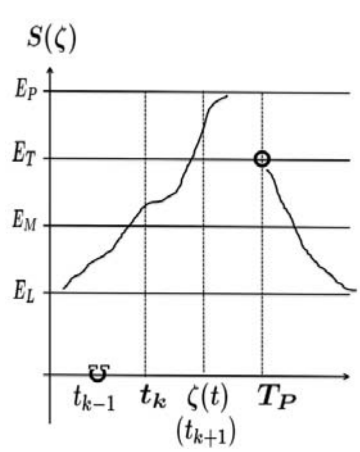

FIGURE 4. States of before and after the beginning of manufacturing start

In Figure 4, start date pursuance probability is defined as

Definition 4.1. *Start date pursuance probability*

$$P_\zeta\{S(\zeta(t_{k+1})) > E_P)\}, \quad \forall t_{k-1} < t (= t_{k+1}), \quad \zeta(t) < T_P(\textit{start date}). \tag{10}$$

Further, start date non-pursuance probability is defined as

Definition 4.2. *Start date non-pursuance probability*

$$P_\zeta\{S(\zeta(t_{k+1})) < E_M\}, \quad \forall t_{k-1} < t, \quad \zeta(t) < T_P(\textit{start date}). \tag{11}$$

Accordingly, when these probabilities are indicated as $W_1(t_{k+1}), W_2(t_{k+1})$, respectively, they can be described as

$$\begin{aligned} W_1(t_{k+1}) &= P_\zeta\{S(\zeta(t_{k+1})) > E_P\} \\ &= \int_{E_P}^{\infty}\int_{E_P}^{\infty} N_{SP}(E)F_{SP}(E)y(x)dxdE \end{aligned} \tag{12}$$

$$\begin{aligned} W_2(t_{k+1}) &= P_\zeta\{S(\zeta(t_{k+1})) \leq E_M\} \\ &= \int_{E_P}^{\infty}\int_{0}^{E_M} N_{SP}(E)F_{SP}(E)y(x)dxdE \end{aligned} \tag{13}$$

where, $N_{SP}(E)$ is a state density, $F_{SP}(E)$ a distribution function according to Fermi-Dirac [8, 9]. $y(\bullet)$ is a probability density function of a standard normal distribution

$$y(x) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}x^2\right). \tag{14}$$

Next, in a state $E_M \leq L \leq E_P$, when start has not been determined yet, a system will register a loss period. In this case, because work start itself has not been determined yet, the whole of such period becomes loss. Now, assuming that its value is C, C is as

$$U(t_{k+1}) = C.$$

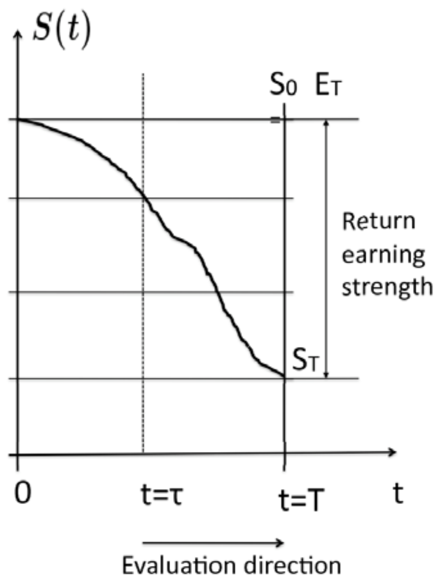

Figure 5. Dynamic behavior of production density function

Here, assuming that C_P; is a pursuance cost, C_M; non-pursuance cost, C_T; a loss cost due to idling, an expectation cost in $[t_k, t_{k+1}]$ (let express it as $= V(t_{k+1})$ will be as

$$V(t_{k+1}) = \frac{C_P W_1(t_{k+1}) + C_M W_2(t_{k+1}) + C_T U(t_{k+1})}{t_{k+1} - t_k}. \tag{15}$$

Here, for the purpose of simplicity, let $C_T \approx 0$. The reason of this is that, in an idling period, production schedule is changed to make the cost be zero. In addition, assuming that, in order to make $V(t_{k+1})$ minimum, $C_M = 0$ as long as T_P is not determined, evaluation for determining T_P simply becomes as

$$|W_1(t_{k+1})| > E_P. \tag{16}$$

Regarding the reason that ζ^* can be converted as $\zeta^* = \ln \zeta$, we have found that, from observed monthly cash flow data (return deviation), a probability density function is log-normally distributed (Figure 5). A theoretical curve was calculated using EasyFit software (http:www.mathwave.com/), and, as a result of Kolmogorov and Smirnov test, the observed values conformed to a log-normal distribution ($P = 0.588$). Parameters of the theoretical curve were $\mu = -0.134$, $\sigma = 0.0873$, $\nu = -0.900$.

Next, a production density function that changes from the initial value of $S(0) = ET$ (flow level), declines while taking positive values. Therefore, as a dynamic model, the following formula is assumed. This model is a frequently used model as a reality-based model. Therefore, by performing conversion of $t = \ln t$, the following assumption is made.

Assumption 1. *Stochastic model of production density function (stochastic differential equation of log-normal type)*

$$\frac{dS(t)}{S(t)} = mdt + \sigma dw^s(t) \tag{17}$$

where m indicates a drift term of $S(t)$, σ a variance of $S(t)$ and $w^s(t)$ is a standard Brownian motion.

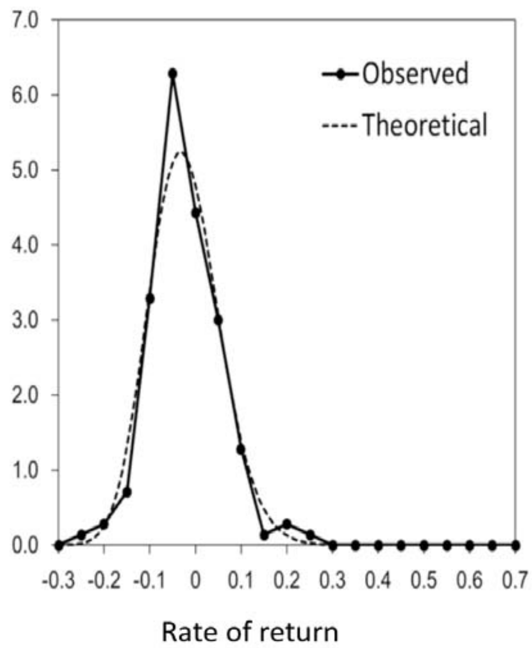

FIGURE 6. Probability density function of rate-of-return deviation: actual data (solid line) and data based on theoretical formula (dotted line)

5. **Dynamic Estimation of Production Process.** Now, as dual problems of this system, return, that is, a cash flow model is formulated [14]. Here, a cash flow function obtained from $S(t)$ is defined as

Definition 5.1. *Cash flow function*

$$P(t) \equiv P(t : S(t)) \tag{18}$$

$$\frac{dP(t)}{P(t)} = mdt + \sigma dw^p(t) \tag{19}$$

where, let $P(0) = P_0$, and $w^P(t)$ is a standard Brownian motion. On this occasion, it is supposed that $P(t)$ can be measured through a certain observation mechanism (refer to Figure 7). Here, Figure 8 indicates an input-output rate control method by a processing line (synchronization of processes). By estimating an expected value and a variance value of throughput of all processes, a manufacturing process is finished with every process finished on time. Here, as a measurement item in a processing line, an "average" and "variance" of throughput is made be measurement data.

According to process progress of a process, the process makes input request to an input side buffer that is the preceding process. In order to keep lead time (throughput) of the process in question strictly, it controls the "line". With a production rate of the thus controlled processing line, output is performed to an output side buffer.

Regarding control of the output side buffer, by receiving output of the "processing line" that is the preceding process by the buffer, the output side buffer controls, for the subsequent processes, an output rate of its own in order to keep the total throughput strictly.

Under such control, it is necessary for a processing line to measure throughput of its own. Therefore, although a throughput function must be obtained by measurement in the input side and output side, it is not always entirely observable (complete) over all processes.

Therefore, taking an average value and variance of a throughput function as a measurable variable, and using the Kalman filter theory, the average value and variance of the throughput function is estimated.

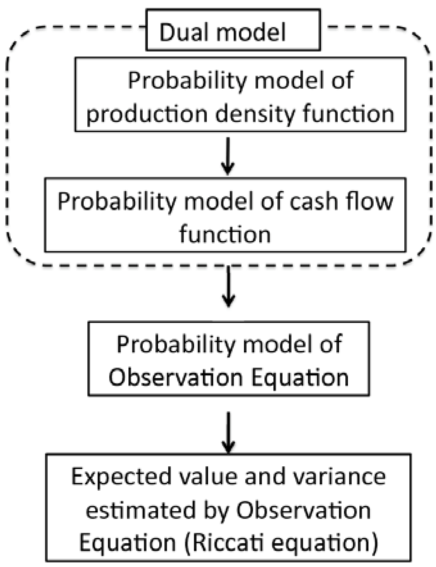

FIGURE 7. Process of expected value and variance value observed from dual model

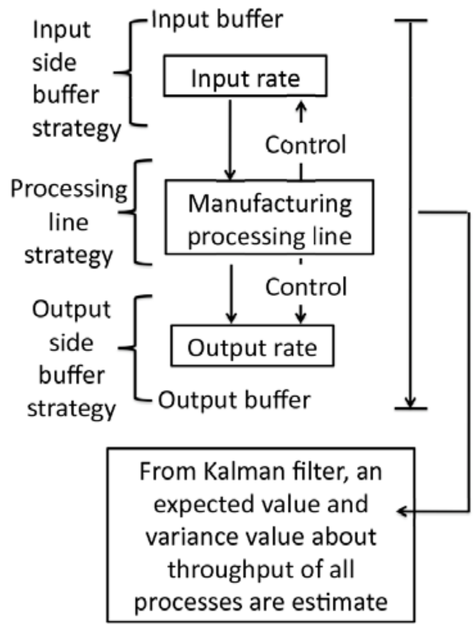

FIGURE 8. Input-output rate control by a processing line (synchronization of processes) and estimation of expected value and variance of throughput of all processes

Next, on the premise of such system model, a method for strictly keeping a manufacturing process in a state where a system is in a non-complete state will be described [15].

Here, assuming that filtration by a measurement process is $\{F_t^{\xi}\}_{t=\mathcal{R}}$,

$$F_t \supset F_t^{\xi}, \quad F_t \neq F_t^{\xi} \tag{20}$$

By this, the system becomes a non-complete model [15].

In Figure 8, let a probability model of a production process be as

$$dw_s(t) = mw_s(t)dt + \sigma_s dw^s(t) \tag{21}$$

where, σ_s is indicates variance of $w^s(t)$, and $w^s(t)$ is a standard Brownian motion.

A probability model of an observation formula is set as

$$\begin{gathered} d\xi(t) = w_s(t)dt + \sigma_\xi dw^\xi(t), \\ \forall t \in \mathcal{R}_t, \quad F_t^\xi \subset F_t, \quad F_t \neq F_t^\xi, \quad \xi(t) \in F_t^\xi \end{gathered} \tag{22}$$

where, σ_ξ indicates variance of $\xi(t)$, and $w^\xi(t)$ a standard Brownian motion. An evaluation formula of a production process $PI(t, \mu(t))$ indicates an estimated cash flow, and satisfies the following formula.

From Kalman filter, an expected value and variance value about throughput of all processes are estimated.

$$PI(t, \mu(t)) = \begin{cases} 0 & \text{Operation stop} \\ E\Big[e^{-rdt}P_1(t+dt, \mu(t)) & \\ +d\xi(t)\Big|F_t^\xi\Big] & \text{In the course of determination} \end{cases} \tag{23}$$

Let define observables (expected value and variance) estimated from Kalman filter by

Definition 5.2. *Observables expected value and variance*

$$\mu(t) = E\Big[w_s(t)|F_t^{\xi}\Big] \tag{24}$$

$$\psi(t) = V\Big[w_s(t)|F_t^{\xi}\Big]. \tag{25}$$

From Kalman filter theory [11], we obtain

$$\begin{aligned} d\mu(t) &= m\mu(t)dt \\ &\quad + \frac{\psi(t)}{\sigma_\xi}\Big\{\frac{1}{\sigma_\xi}(d\xi(t) - \mu(t)dt)\Big\} \end{aligned} \tag{26}$$

$$d\psi(t) = \Big[-2m\psi(t) + \sigma_s^2 - \frac{\psi^2(t)}{\sigma_t^2}\Big]dt. \tag{27}$$

Formula (26) and Formula (27) are equations referred to as a Riccati type equation, and can derive an analytical solution.

Here, an evaluated value $PI(t, \mu(t))$ of a cash flow is a cash flow function estimated by Kalman filter, and is given by

$$rPI(t, \mu(t))dt = E_t[dPI(t, \mu(t))] \tag{28}$$

where, let assume that $\mu(t)$ has a constrained condition of $\mu_L \le \mu(t) \le \mu_H$.

In this regard, however, r is a risk-free rate. At this time, Formula (28) means

$$\begin{aligned} &[\text{Risk-free rate}] \times [\text{Evaluation value}] \\ &= [\text{Time flow for evaluated value}]. \end{aligned} \tag{29}$$

Also, $PI(t, \mu(t))$ indicates the current cash flow evaluated value for estimated $\mu(t) = \mu_L$. The reason is that, although essentially it is indicated by $PI(t, \mu(t), \psi(t))$, because $\psi(t)$ is specified uniquely, it is treated as

$$PI(t, \mu(t)) \equiv PI(t, \mu(t), \psi). \tag{30}$$

Here, at the critical value lower limit of expected value of a cash flow $\mu(t) = \mu_L$, a condition to continue manufacturing can be expressed as

$$\mu_L = \frac{\partial PI(t, \mu(t))}{\partial \mu(t)}\Big|_{\mu(t)=\mu_L}, \quad \forall \mu(t) \equiv E_t[w_s(t)|F_t^{\xi}]. \tag{31}$$

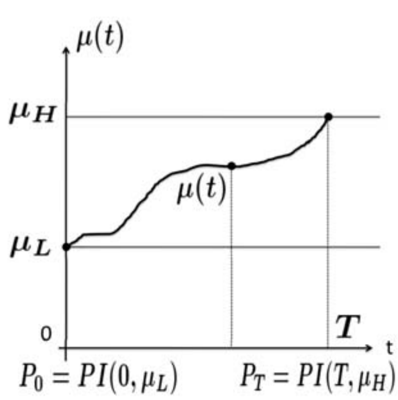

Figure 9. Cash flow evaluated value for estimated expected value

Now, when Ito's lemma [14] is applied to $PI(t,\mu(t))$, we obtained

$$dPI(t,\mu(t)) = \frac{1}{2}\sigma_\xi^2\mu(t)^2\frac{\partial^2 PI(t,\mu(t))}{\partial\mu(t)^2}dt + m\mu(t)\frac{\partial PI(t,\mu(t))}{\partial\mu(t)}dt + \sigma_\xi\mu(t)\frac{\partial PI(t,\mu(t))}{\partial\mu(t)}dw^\xi(t). \tag{32}$$

From this, we obtain

$$[rPI(t,\mu(t))]dt = \frac{1}{2}\sigma_\xi^2\mu^2(t)\frac{\partial^2 PI(t,\mu(t))}{\partial\mu(t)^2}dt + m\mu(t)\frac{\partial PI(t,\mu(t))}{\partial\mu(t)}dt + \sigma_\xi\mu(t)\frac{\partial PI(t,\mu(t))}{\partial\mu(t)}E_t[dw^\xi(t)]. \tag{33}$$

However, because $E_t[dw^\xi(t) = 0]$, in order for Formula (33) to hold, the following formula needs to hold.

$$\frac{1}{2}\sigma_\xi^2\mu^2(t)\frac{\partial^2 PI(t,\mu(t))}{\partial\mu(t)^2} + m\mu(t)\frac{\partial PI(t,\mu(t))}{\partial\mu(t)} - rPI(t,\mu(t)) = 0 \tag{34}$$

Further, let Formula (31) be applied to Formula (34). In this regard, however, let $PI(t,\mu(t))$ be the following formula [14].

$$PI(t,\mu(t)) = A\mu^\beta(t) + \frac{\mu(t)}{\nu} - \frac{\mu_L}{r}, \quad \nu = r - m \tag{35}$$

From this, by substituting Formula (35) into Formula (33), the characteristic equation is obtained as

$$\frac{1}{2}\sigma_\xi^2\beta^2 + \left(\nu - \frac{1}{2}\sigma_\xi^2\right)\beta - r = 0, \quad \forall\beta < 0. \tag{36}$$

Therefore, β is obtained by

$$\beta = \frac{1}{\sigma_\xi^2}\left[\left\{\frac{1}{2}\sigma_\xi^2 - \nu\right\} + \left[\left\{\frac{1}{2}\sigma_\xi^2 - \nu\right\}^2 + 2\sigma_\xi^2 r\right]^{1/2}\right]. \tag{37}$$

In addition, from $\left.\frac{\partial PI(t,\mu(t))}{\partial\mu(t)}\right|_{\mu(t)=\mu_L} = 0$, we obtain

$$A\beta\mu^{\beta-1}(L) + \frac{1}{\nu} = 0. \tag{38}$$

Therefore, P_I follows

$$PI(t,\mu(t)) = \left[\frac{\mu(t)}{\nu} - \frac{\mu_L}{r}\right] - \beta^{-1}\cdot\mu^{-\beta}(L)\cdot\left[\frac{\mu_L}{\nu}\right]\mu^\beta(t) = \left[\frac{\mu(t)}{\nu} - \frac{\mu_L}{r}\right] - \left[\frac{\mu_L}{\beta}\right]\cdot\left[\frac{\mu(t)}{\mu_L}\right]^\beta. \tag{39}$$

Further, assuming that a current evaluated value of a cash flow at $t = \tau$ is V_t, V_t follows

$$V_t = E_t\left[\int_t^\infty PI(t,\mu(\tau))\cdot e^{-\kappa\tau}d\tau\right]. \tag{40}$$

Now, let an evaluated value at $t \in [0, T]$ be expressed as

$$k_{PI} = E_t \left[\int_0^T PI(t, \mu(\tau)) \cdot e^{-\kappa\tau} d\tau \right]. \tag{41}$$

In this regard, however, because $PI(t, \mu(\tau))$ is an actual average value of a cash flow, the following formula holds.

$$PI(t, \mu_H) \cong PI(t, \mu_L) + k_{PI} \tag{42}$$

Therefore, from Formula (39) and Formula (42), a condition of $\mu(t)$ can be derived. That is, it is as

$$\left[\frac{\mu_H - \mu_L}{\nu}\right] - \left[\frac{\mu_L}{\beta}\right] \left\{ \left[\frac{\mu_H}{\mu_L}\right]^\beta - 1 \right\} = k_{PI}. \tag{43}$$

From Formula (43), the following formula is obtained.

$$\left[\frac{\mu_H - \mu_L}{\nu}\right] - k_{PI} = \left[\frac{\mu_L}{\beta}\right] \left\{ \left[\frac{\mu_H}{\mu_L}\right]^\beta - 1 \right\} \tag{44}$$

Moreover, from Formula (44), we obtain

$$\left[\frac{\mu_H}{\mu_L}\right]^\beta = \left[\frac{\beta}{\mu_L}\right] \left\{ \frac{(\mu_H - \mu_L)}{\nu} - k_{PI} \right\} + 1. \tag{45}$$

In order for Formula (45) to become a positive value, the following formula must hold.

$$\frac{\beta\mu_H - (\beta - \nu)\mu_L}{\nu\beta} > k_{PI}, \qquad \forall \nu = r - m,\ \alpha,\ \beta,\ k_{PI} > 0,\ \mu_H > \mu_L > 0 \tag{46}$$

Therefore, because $\beta < 0$, $\nu > 0$. $\mu_H > 0$, $\mu_L > 0$, $k_{P_1} > 0$, a condition satisfying equivalence of a value becomes as

$$\beta\mu_H - (\beta - \nu)\mu_L < 0. \tag{47}$$

At this time, $PI(t, \mu(t))$ is given by Formula (37) and Formula (39). Further, $\mu(t)$ can be find from the simultaneous differential equation of Formula (26) and Formula (27). In this regard, however, initial values $\mu(0)$ and $\psi(0)$ are given by

$$\begin{aligned} \mu(0) &= E_t[w_s(0) | F_0^\xi] \\ \psi(0) &= V_t[w_s(0) | F_0^\xi]. \end{aligned} \tag{48}$$

Next, let an expected throughput rate be R. Meanwhile, throughput of a manufacturing process means a drift term m of Formula (19) indicating a probability model of a cash flow.

Now, assuming that a risk premium is r_p, let relation between a current cash flow evaluated value and a cash flow setting desired value N be defined as

Definition 5.3. *Cash flow setting desired value N*

$$N = PI(t, \mu(t)) \left[\frac{r + r_p - m}{R - m}\right]. \tag{49}$$

In this regard, however, a risk premium is something like interest according to risk, and it is supposed that it is already known. From this, a throughput rate R to be found can be calculated as

$$R = m + PI(t, \mu(t)) \left[\frac{r + r_p - m}{N}\right] \tag{50}$$

where, $PI(t, \mu(t))$ is expressed by

$$PI(t, \mu(t)) = \left[\frac{\mu(t)}{\nu} - \frac{\mu_H}{r}\right] - \left[\frac{\mu_L}{\beta}\right] \cdot \left[\frac{\mu(t)}{\mu_L}\right]^{\beta}. \quad (51)$$

That is, R of Formula (50) indicates an expected throughput for which a condition of the upper limit setting value must be satisfied for each estimated throughput of a system. In other words, it is an expected value of throughput indicating a judgment criterion for performing at start date set in conformity to a parameter condition of the system.

6. **Numerical Result.** Expected throughput R for an estimated expected value $\mu(t)$ of a cash flow and numerical value data of an evaluated value is cited here. Table 1 shows parameters used for creating a graph of expected throughput R for estimated expected value $\mu(t)$ of a cash flow. It seems that the upper limit value have a large influence on Type 1, Type 2 and Type 3.

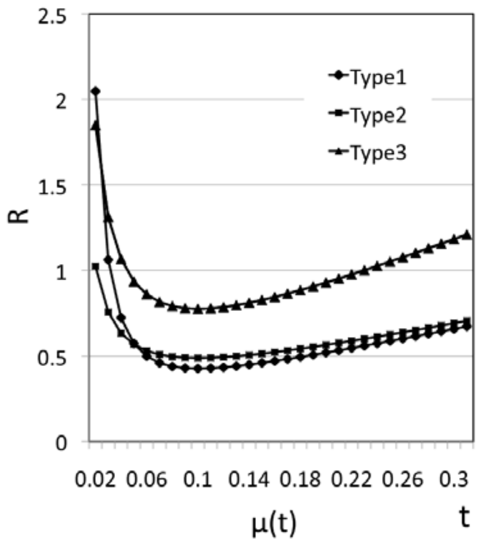

FIGURE 10. Expected throughput rate R for estimated expected value $\mu(t)$ of cash flow

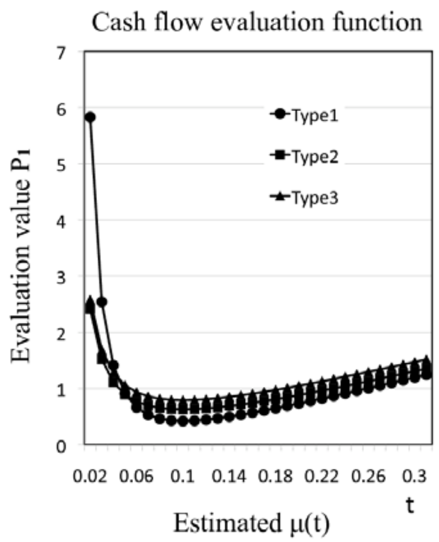

FIGURE 11. Cash flow evaluated value PI for estimated expected value $\mu(t)$ of cash flow

Table 1. Set parameter values

Expected throughput rate for estimated expected value $\mu(t)$ of cash flow			
	Type 1	Type 2	Type 3
Volatility	0.25	0.3	0.3
Input amount setting	1	1	1
Stable throughput	0.5	0.5	0.5
Throughput	0.3	0.3	0.3
Upper limit	0.2	0.3	0.6
Under limit	0.1	0.1	0.1

Table 2. Set parameter values

Cash flow evaluated value for estimated expected value $\mu(t)$ of cash flow			
	Type 1	Type 2	Type 3
Volatility	0.25	0.3	0.3
Input amount setting	1	1	1
Stable throughput	0.5	0.5	0.5
Throughput	0.3	0.3	0.3
Upper limit	0.2	0.3	0.6
Under limit	0.1	0.1	0.1

On the other hand, Table 2 shows parameters used for creating a graph of an evaluation function for estimated expected value $\mu(t)$ of a cash flow. On Type 1, Type 2, Type 3, the upper limit value seems to have a large influence.

That is, it means that both of expected throughput R and an evaluated value are being constrained by the upper and lower limit values, and that influence of these constraint conditions is large. At the time of system design, it is important to set these parameters to be a realistic value.

7. Conclusion.

In the present paper, we tried applying an energy level known in physics to a manufacturing process of a production system. This is a way of thinking that has not existed in the conventional ideas. Based on this idea, we calculated throughput of manufacturing by classifying manufacturing processes into a complete state and non-complete state. Even in a incomplete information state, a manufacturing throughput rate can be estimated by utilizing Kalman filter. About this uncertain state, a manufacturing throughput was confirmed by numerical value calculation.

Acknowledgment. We thank Dr. E.Chikayama, Associate professor of Niigata University of International and Information Studies, for verifying the log-normal distribution type data.

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