International Journal of Innovative Computing, Information and Control
Volume **9**, Number **12**, December **2013**

ICIC International ©2013 ISSN 1349-4198
pp. 4917–4930

IMPROVING THROUGHPUT BY CONSIDERING THE PRODUCTION PROCESS

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Received December 2012; revised April 2013

Abstract. *In this study, a delay of process throughput makes it clear that it is due to volatility. Moreover, to understand the difference between the asynchronous method, which causes a delay in the manufacturing process, and the synchronous method, which reduces the process throughput time in manufacturing processes, we manufactured equipment. "Synchronization with preprocess" is a manufacturing method used to increase throughput. Because synchronization reduces volatility from the start of production until it finishes, it is the best method available. However, it is difficult in real-world situations to prepare all workers who can work within the target working time in the process. Then, we propose a realistic method termed "Synchronization with preprocess". "Synchronization with preprocess" means that by carrying out the reclassification of the working process, it is a method for smoothing the volatility of the working time. To verify our proposed method, we show that by using synchronization to reduce volatility, we decrease risk. Here, for example, a risk means the delay of working in each process.*
Keywords: Synchronization with preprocess, Throughput, Volatility, Stochastic differential equation of log-normal type, Production flow process

1. **Introduction.** Several studies have addressed the problem of increasing the productivity of production processes used in the manufacturing industry [1, 2]. Moreover, in the field of manufacturing, various theories have been applied to improve and reform manufacturing processes and increase productivity. In a previous study [3], we addressed the problem of reducing construction work and inventory in the steel industry. Specifically, we investigated the relationship between variations in the rate of construction and delivery rate. In this study, we perform analysis using the queuing model and apply log-normal distribution to model the system in the steel industry [3].

Moreover, several studies have reported approaches that lead to shorter lead times [4, 5]. From order products, lead time occurs on the work required preparation of the members for manufacturing.

Many aspects can potentially affect lead time. For example, from order products, the lead time from the start of development to the completion of a product is called the time-to-finish time, such as the work required preparation of the members for production equipment.

Moreover, several studies have focused on reducing customer lead times. In [6], the author addresses the problem of reducing the production lead time [6].

In [7], the authors propose a method that increases both production efficiency and production of a greater diversity of products for customer use. Their proposed approach results in shortened lead times and reduces the uncertainty in demand. Their method captures the stochastic demand of customers and produces solutions by solving a nonlinear stochastic programming problem.

In summary, several studies have considered uncertainty and proposed practical approaches to shorten the lead time. The demand is treated as a stochastic variable and applies mathematical programming. To our knowledge, previous studies have not treated lead time as a stochastic variable.

Because fluctuations in the supply chain and market demand and the changes in the production volume of suppliers are propagated to other suppliers, their effects are amplified. Therefore, because the amounts of stock are large, an increase or decrease of the suppliers' stock is modeled using a differential equation. This differential equation is said as Billwhip model, represents a stock congestion [8, 9].

The theory of constraints (TOC) describes the importance of avoiding bottlenecks in production processes [10]. When using manufacturing equipment, delays in one production step are propagated to the next. Hence, the use of manufacturing equipment may lead to delays. In this study, we apply a physical approach and regard each step as a continuous step. By applying this approach, we can mathematically analyze the delay of each step and obtain methods to address it. To the best of our knowledge, previous studies have not applied physical approaches to analyze delays.

In a previous study [11], we constructed a state in which the production density of each process corresponds to the physical propagation of heat [15]. Using this approach, we showed that a diffusion equation dominates the manufacturing process.

In other words, when minimizing the potential of the production field (stochastic field), the equation, which is defined by the production density function $S_i(x,t)$ and the boundary conditions, is described using the diffusion equation with advection to move in transportation speed ρ. The boundary conditions mean a closed system in the production field. The adiabatic state in thermodynamics represents same state [11].

To achieve the goal of a production system, we propose using a mathematical model that focuses on the selection process and adaptation mechanism of the production lead time. We model the throughput time of the production demand/manufacturing system in the manufacturing stage by using a stochastic differential equation of log-normal type, which is derived from its dynamic behavior. Using this model and the risk-neutral integral, we define and compute the evaluation equation for the compatibility condition of the production lead time. Furthermore, we apply the synchronization process and show that the throughput of the manufacturing process is reduced [12, 13].

In this paper, we describe the differences between the synchronous and asynchronous models and show that the throughput of a manufacturing process depends on volatility. Synchronization implies that the machines and assembly lines manufacture the required production volumes in accordance with timing requirements. Moreover, to understand the difference between the asynchronous method, which causes a delay in the manufacturing process, and the synchronous method, which reduces the process throughput time in manufacturing processes, we manufactured equipment.

"Synchronization with preprocess" is a manufacturing method used to increase throughput. Synchronization reduces volatility from the start of production until it finishes. The automotive industry has adopted the synchronization process to reduce volatility.

We show that by using our proposed physical approach, we can obtain results similar to those obtained by the synchronization process.

The synchronization process is the best method available. However, because it is difficult to apply in real-world situations, we propose a realistic method termed "Synchronization with preprocess". "Synchronization with preprocess" means that by carrying out the reclassification of the working process, it is a method for smoothing the volatility of the working time. In general, the lead times of processes should be set equal to the same value. However, in the "Synchronization with preprocess" method, before starting the manufacturing process, we analyze a particular process and select different lead times. Using this approach, the "Synchronization with preprocess" method can achieve a much better total throughput. To the best of our knowledge, the "Synchronization with preprocess" method has not been previously proposed.

To verify our proposed method, we apply it to a flow production system. We prove that our proposed method reduces volatility, which in turn reduces risk. Here, for example, a risk means the delay of working in each process.

2. **Production Systems in the Manufacturing Equipment Industry.**

The production methods used in manufacturing equipment are briefly covered in this paper. More information is provided in our report [12]. This system is considered to be a "Make-to-order system with version control", which enables manufacturing after orders are received from clients, resulting in "volatility" according to its delivery date and lead time. In addition, there is volatility in the lead time, depending on the content of the make-to-order products (production equipment).

In Figure 1(A), the "Customer side" refers to an ordering company and "Supplier (D)" means the target company in this paper. The product manufacturer, which is the source of the ordered manufacturing equipment presents an order that takes into account the market price. In Figure 1(B), the market development department at the customer's factory receives the order through the sale contract based on the predetermined strategy.

3. **Manufacturing Process Model.**

It is often represented by a log-normal distribution [3]. The sales figure for the probability density function of the rate of return shows the log-normal distribution in Figure 2. Because small-to-midsize firms often do not have enough working capital, to sustain company operations, they are forced to raise working capital from financial institutions. It is non-linear in the case such as the products of different product specifications with fluctuations in demand or multi-kind small lot. We will report about this separately.

Thus, if the rate of return follows a log-normal distribution, we can assume that the cash flow will also follow the same log-normal distribution. Therefore, a cash flow model is defined as follows [14].

Definition 3.1. *Definition of a cash flow model*

$$\frac{dQ(t)}{Q(t)} = \mu dt + \sigma dW^Q(t) \tag{1}$$

where $Q(t)$ is an expected money amount of production for each month. The left-hand side is a monthly rate of return, and a rate of return varies with expected value μ. Further, σ represents a volatility, and $W^Q(t)$ standard Brownian motion.

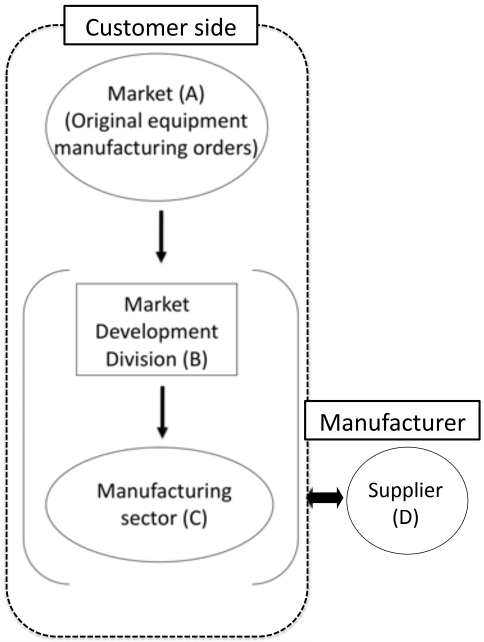

Figure 1. Business structure of company of research target

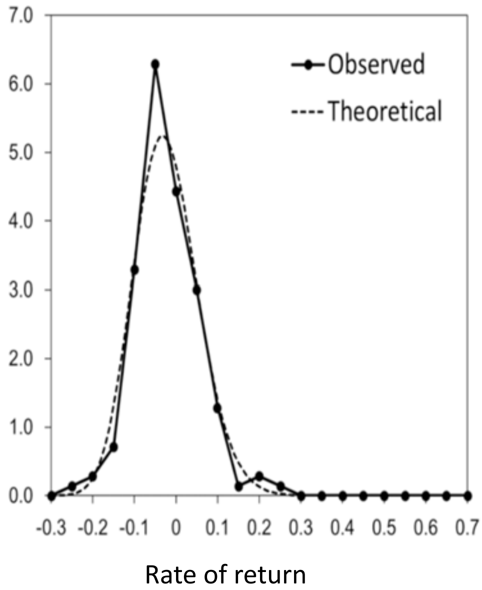

Figure 2. Probability density function of rate-of-return deviation: actual data (solid line) and data based on theoretical formula (dotted line)

From the data of monthly rate of return observed, its probability density function was calculated (Figure 2). As a result, it was found that the probability density function conforms to log-normal distribution (Figure 2, Theoretical).

Theoretical curve was calculated using EasyFit software (http://www.mathwave.com/), and as a result of Kolmogorov and Smirnov test, the observed values conformed to a log-normal type probability density function. Because, in the goodness-of-fit test of Kolmogorov-Smirnov, a null hypothesis that it is "log-normal" was not rejected with rejection rate 0.2, this data conforms to "log-normal" distribution. *P-value* was 0.588. The parameters of a theoretical curve were: $\mu_p = -0.134$ (average), $\sigma_p = 0.0873$ (standard deviation), $\gamma_p = -0.900$. The theoretical curve is given by the following formula.

$$f(x) = \frac{1}{\sqrt{2\pi}(x-\gamma_p)\sigma_p} \exp\left\{-\frac{1}{2}\left(\frac{(\ln x - \gamma_p) - \mu_p}{\sigma_p}\right)^2\right\} \tag{2}$$

We assumed manufacturing process follows a log-normal probability distribution. In fact, we found to be the log-normal probability distribution by analyzing the rate of return on monthly data of manufacturing operations (1999/1 to 2008/12) over the past 10 years (Figure 2 reference). We think a rate of return is proportional to the manufacturing process lead time.

4. **Production Flow Process.** Figure 3 depicts a manufacturing process that is termed as a production flow process. This manufacturing process is employed in the production of control equipment. In this example, the production flow process consists of six stages. In each step S1-S6 of the manufacturing process, materials are being produced.

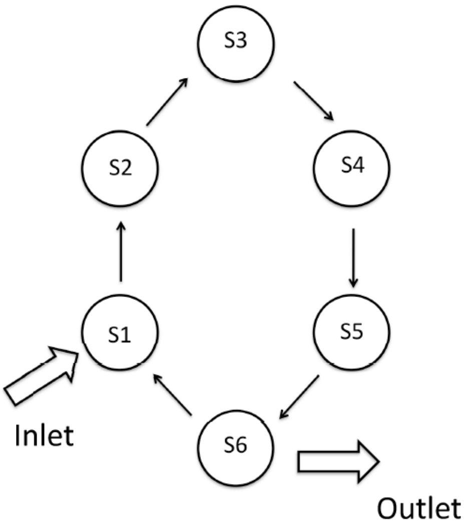

FIGURE 3. Production flow process

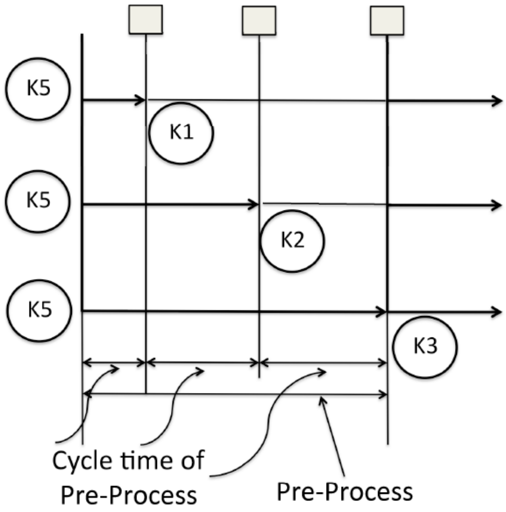

FIGURE 4. Preprocess in manufacturing equipment

The direction of the arrows represents the direction of the production flow. In this process, production materials are supplied through the inlet and the end-product is shipped from the outlet. For this flow production system, we make the following two assumptions.

4.1. **Synchronous model.**

Definition 4.1. *The role of the synchronization model is to reduce the process throughput, i.e.,*

$$dS(t,x) = rS(t,x)dt + \sigma S(t,x)dW(t) \tag{3}$$

where $S(t,x)$ represents the production density function as a function of the synchronous status.

Synchronization minimizes the risk in the production process. To realize synchronization, we set the throughput of each stage to the same value. Because we set the working time for the workers in each work stage, there is no volatility in the working time between processes.

Here, $S(t,x)$ represents the production density function as a function of the synchronous status when the equipment is manufactured. t represents the manufacturing time. x represents the production process term when products are manufactured continuously. σ represents the volatility at each stage, and $W(t)$ represents the Wiener process.

4.2. **Asynchronous model.**

Definition 4.2. *When we use the asynchronous model to represent a dynamical system, the throughput is not reduced.*

$$d\tilde{S}(t,x) = \bar{C}(t,x)\tilde{S}(t,x)dt + \tilde{\sigma}\tilde{S}(t,x)dW(t) \tag{4}$$

where $\bar{C}(t,x)$ represents the average working time of the total processes when the equipment is manufactured using an asynchronous process.

$$\bar{C}(t,x) = E[C(t,x)] = E\left[\sup_{t\in[0,T]} ||C(t,x)||^p\right] < \infty, \quad p > 2 \tag{5}$$

where, $C(t,x)$ exists uniquely. Therefore, it is clear that Equation (5) is established. $C(t,x)$ is the arbitrage-free term under the equivalent martingale measure.

Therefore, each stage of the production flow process can be represented by the Wiener process. Because, the working time in each stage fluctuate stochastically. Then, the relative production density $\tilde{S}(t,x)$ *is expressed as follows* [14]*:*

$$\tilde{S}(t,x) = \tilde{S}(0,x) - \int_0^t \tilde{S}(u,x)\sigma_u^* \mathbf{1}_{T-u} d\hat{W}(t) \tag{6}$$

That is, the volatility σ_u^* *exists. Then* $\tilde{S}(t,x)$ *is*

$$\tilde{S}(t,x) = \frac{S(t,x)}{S(t,0)} \exp\left\{\int_0^t r_u du - \int_0^t C(t,u)du\right\} \tag{7}$$

Definition 4.3. *According to the asynchronous model, the average time of working is as follows:*

$$r_u^c \equiv r_u - E[C(t,x)] \tag{8}$$

From Equation (8), we obtain

$$\tilde{S}(t,x) = \frac{S(t,x)}{S(t,0)} \exp\left\{\int_0^t r_u^c du\right\} \tag{9}$$

where $\tilde{S}(t,x)$ represents the production density of the asynchronous model. In the asynchronous model, the production workers at each stage do not complete the assigned work within the allocated time period. Therefore, at each stage, there is a volatility in the working time.

The solution of Equation (3) is

$$S(t,x) = S(0,x) \exp\left\{\left(r - \frac{1}{2}\sigma^2\right)t + \sigma W(t)\right\} \tag{10}$$

where r indicates the total average working time when manufacturing using a synchronous process.

According to Equation (10), the production density $\tilde{S}(t,x)$ of the asynchronous model is as follows:

$$\tilde{S}(t,x) = \tilde{S}(0,x) \exp\left\{\left(r_u^c - \frac{1}{2}\sigma_c^2\right)t + \sigma_c \hat{W}(t)\right\} \tag{11}$$

where from Girsanov theorem, $\hat{W}(t)$ is

$$\hat{W}(t) = W(t) + \int_0^t \lambda(u)du \tag{12}$$

Therefore, according to Equations (5) and (6), the solution of $\tilde{S}(t,x)$ is as follows (Asynchronous model):

$$\tilde{S}(t,x) = \tilde{S}(t,0) \exp\left\{\left(r_c - \frac{1}{2}\sigma_c^2\right)t + \sigma_c W(t)\right\} \tag{13}$$

$$d\tilde{S}(t,x) = r_c\tilde{S}(t,x)dt + \sigma_c\tilde{S}(t,x)d\hat{W}(t) \tag{14}$$

$\tilde{S}(t,x)$ is a martingale with respect to F_t [14].

Therefore, $\tilde{S}(t,x)$ satisfies Equation (4) (Asynchronous model).

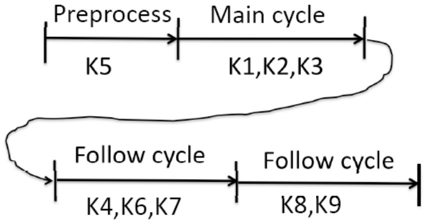

FIGURE 5. "Synchronization with preprocess" method in manufacturing equipment

4.3. **Synchronization with preprocess.**

Definition 4.4. *To reduce the risk of increasing the processing time, we adopt the "Synchronization with preprocess" method for reducing the risk of increasing the processing time.*

In Figure 5, the preprocess represents the work performed until the actual process begins. We execute this preprocess in advance to eliminate the idle time after the classification of the processes. For instance, in Figure 5, the preprocess represents the termination of the operation of step K5. By assigning step K5 to be the preprocess, there are eight remaining processes. Of the three cycles performed in Figure 5, the first cycle is $\{K1, K2, K3\}$, the second cycle is $\{K4, K6, K7\}$, and the third cycle is $\{K8, K9\}$. After the completion of the third cycle, the workers begin manufacturing the next product. Hence, the first manufacturing process initiates the first cycle. By adopting the preprocess cycle, the third cycle can be performed in parallel.

5. **Results of Test-run.**

5.1. **Result of Test-run1.** Test-run1 is an asynchronous process. Therefore, the throughput at each step of Test-run1 is different, the throughput of the entire stage becomes stochastic. Moreover, the stochastic throughput, which is a function of the current time and time remaining until the end of the stage, affects the performance of the entire system. In Tables 2 and 3, we present data that validates our findings presented above.

Therefore, the ratio of the measured throughput to the target throughput is considered as the drift term r_u^c in Equation (14). The fluidity of the system is affected by the throughput at each stage. In other words, because the manufacturing progress is affected by bottlenecks, the drift term r_u^c can be defined using the stochastic throughput (Equations (9)-(11)).

Here the drift term r_u^c is

$$r_u^c = \frac{4.4}{6}\ (0.73) \tag{15}$$

$$r_u^c = \frac{5.5}{6}\ (0.92) \tag{16}$$

The required theoretical throughput for six pieces of equipment/day is computed in Equation (15). However, the actual throughput corresponds to 4.4 pieces of equipment/day.

Furthermore, we can use the same approach to compute the volatility of the throughput at each stage. This average of volatility is given as follows:

$$\sigma_s \approx 0.29 \left(= \frac{1}{N} \sum_{i=1}^{N} \sigma^i(x) \right) \tag{17}$$

Therefore,

Definition 5.1. *The system throughput in this model (Production evaluation model)*

$$d\tilde{S}(t,x) = 0.73\tilde{S}(t,x)dt + 0.29\tilde{S}(t,x)d\hat{W}(t) \tag{18}$$

5.2. **Result of Test-run2.** Next, we consider the case of Test-run2.

Test-run2 is synchronous process. In this case, the process is set in such a way that each stage has the same throughput. Therefore, no risks are introduced as the process progresses. Hence, in principle, the throughput at each stage satisfies the condition. Moreover, because the manufacturing processes require synchronization, we can easily define the "synchronization throughput".

This system has essentially no risk. However, in Tables 4 and 5 we do not observe any values of volatility equal to zero. Therefore, in Equation (3), the term σ is equal to the average volatility.

Here, r, σ in Equation (3) are

$$r^1 = \frac{5.5}{6} = 0.92$$
$$r^2 = 1 - 0.06 = 0.94$$

r^1 and r^2 are not much different. The volatility is

$$\sigma = 0.06$$

Therefore, the throughput model of this system is defined as follows.

Definition 5.2.

$$dS(t,x) = 0.92S(t,x)dt + 0.06S(t,x)dW(t) \tag{19}$$

If the system approaches the synchronization, $\sigma \to 1$. If $\sigma \to$ small data ($\sigma = 0.01$), this system becomes stationary.

For the case of a fully synchronized system, see Figure 6. In Figure 7, it shows the integrated finite number of processing stages progress depending on the synchronization throughput of each stage (stationary system).

Specifically, the synchronous production system is the principle, and the processing stages progress in a cycle, i.e., we set the throughput at T_1, T_2 and T_3 in Figure 8, and synchronize the stages in a cycle.

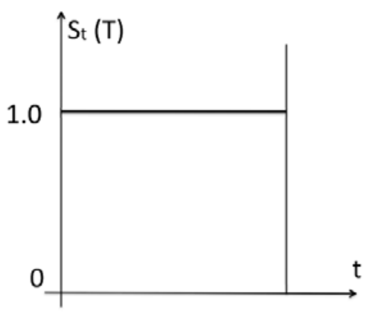

FIGURE 6. Perfect synchronization system

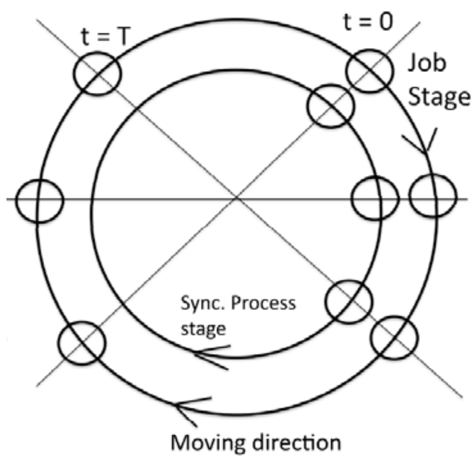

FIGURE 7. Perfect synchronization system

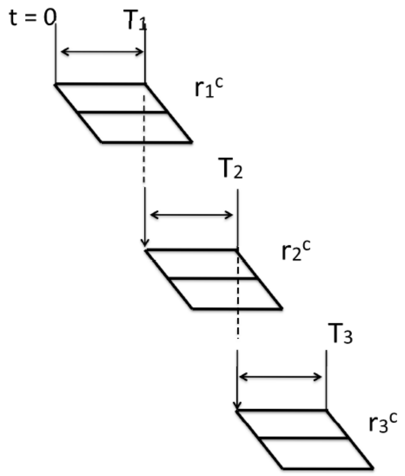

FIGURE 8. Cyclic synchronization

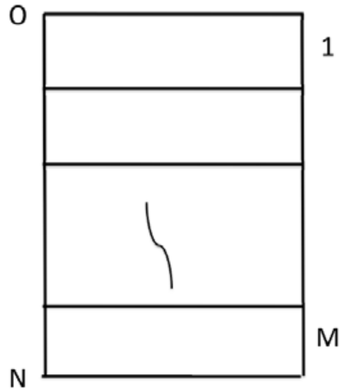

FIGURE 9. Concept of number of M cycle

If Equation (20) is satisfied,

$$\frac{1}{N}\sum_{i=1}^{N} r_i^c \leq \sup\ r_i^c:\ (i = 1, 2, \cdots, N) \tag{20}$$

A risk reduction system was constructed, where $N = kM$ $(k = 1, 2, \cdots, N)$ (k is a positive integer). Because we set the working time for the workers in each work stage, there is no volatility in the working time between processes.

Next, we applied the throughput model and used the results of the test runs to perform numerical calculations. Our model shows that the throughput for each process at each stage is satisfied. If Equation (21) is satisfied, r_i^c $(i = 1, 2, \cdots, N)$ is a real number. This process is a type of bottleneck synchronization. The bottleneck synchronization means a recommendation from the famous "The theory of constraints (TOC)" [10].

1. If $r_i^c \neq r_j^c$, $i \neq j$, synchronization of every stages.
2.

$$\frac{1}{N}\sum_{i=1}^{c} r_i^c \leq \sup\ r_i^c,\ \ i = 1, 2, \cdots, N \tag{21}$$

   if Equation (21) is satisfied, the process is a type of bottleneck synchronization.
3. $r_i^c = r_j^c$, $i = j < N$, the synchronization of some stages.

Here Figure 8 can be considered for item 3.

5.3. **Result of Test-run3.** Next, we consider the case of Test-run3.

Test-run3 represents a "Synchronization with preprocess" method. To achieve the synchronization process by advancing the work before starting the cycle process as a preprocess, the process throughput can be reduced as a result.

As shown in Table 1, even in the case of the "Synchronization with preprocess" method, the volatility of the synchronization throughput ranges from 0.03–0.06. With respect to the overall throughput, volatility is reduced and the risk associated with the process is smaller.

Therefore, to reduce the risk of increasing the processing time, we adopt the "Synchronization with preprocess" method. Below, we present the results from an experiment conducted using $N = 9$, $M = 3$ (cycle) and $k = 3$ (workers).

Table 1. Correspondence between the table labels and the Test-run number

	Table Number	Production process	Working time	Volatility
Test-run1	Table 2	Asynchronous process	627 (min)	0.29
Test-run2	Table 4	Synchronous process	500 (min)	0.06
Test-run3	Table 6	"Synchronization with preprocess" method	470 (min)	0.03

Table 2. Total manufacturing time at each stages for each worker

	WS	S1	S2	S3	S4	S5	S6
K1	15	20	20	25	20	20	20
K2	20	22	21	22	21	19	20
K3	10	20	26	25	22	22	26
K4	20	17	15	19	18	16	18
K5	15	15	20	18	16	15	15
K6	15	15	15	15	15	15	15
K7	15	20	20	30	20	21	20
K8	20	29	33	30	29	32	33
K9	15	14	14	15	14	14	14
Total	145	172	184	199	175	174	181

Table 3. Volatility of Table 2

K1	1.67	1.67	3.33	1.67	1.67	1.67
K2	2.33	2	2.33	2	1.33	1.67
K3	1.67	3.67	3.33	2.33	2.33	3.67
K4	0.67	0	1.33	1	0.33	1
K5	0	1.67	1	0.33	0	0
K6	0	0	0	0	0	0
K7	1.67	1.67	5	1.67	2	1.67
K8	4.67	6	5	4.67	5.67	6
K9	0.33	0.33	0	0.33	0.33	0.33

Using this model, some risks are presented in each cycle. The stochastic throughput is as follows:

$$\partial C_t(x) = \left[\frac{\partial}{\partial x}C_t(x) + D\frac{\partial^2}{\partial x^2}C_t(x)\right]\partial t + \sum_{i=1}^{5}\sigma_t^i(x)dW_t^u(x) \tag{22}$$

$C_t(x)$ is a martingale with respect to F_t. We obtain Equation (23).

$$E\left[\sup_{t\in[0,T]} ||C_t(x)||^p < \infty\right], \quad p \geq 2 \tag{23}$$

In the "Synchronization with preprocess" method, if $C_t(x)$ exists, we can compute the stage synchronization throughput r_t^c. If σ_s^* can be computed, we can use the measured value of the throughput to evaluate the overall processes at $t = T$ as follows.

Definition 5.3. *Evaluation of the relative production density function $\tilde{S}_T(x)$ at $t = T$.*

$$d\tilde{S}(T,x) = r_t^c\tilde{S}(T,x)dt + \sigma_s^*\tilde{S}(T,x)d\hat{W}_t \tag{24}$$

In this case, the reduction of σ_s^* is a key point of building the system. Therefore, we named to "Synchronization with preprocess" method as to reduce this σ_s^*.

6. Analysis of the Test-run Results.

- (Test-run1): Because the throughput of each process (S1-S6) is asynchronous, the overall process throughput is asynchronous. In Table 2, we list the manufacturing time (min) of each process. In Table 3, we list the volatility in each process performed by the workers. Finally, Table 2 lists the target times. The theoretical throughput is obtained as $3\times199+2\times15 = 627$ (min). In addition, the total working time in stage

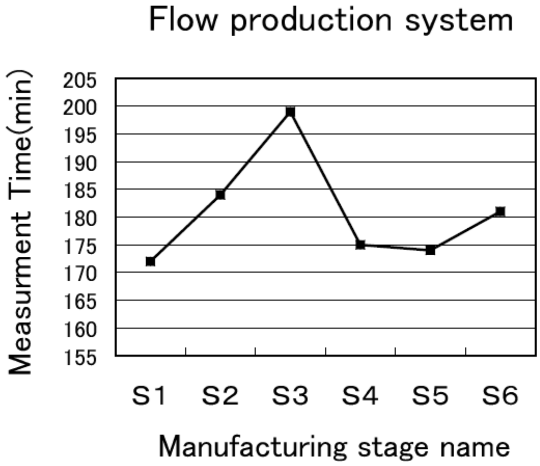

FIGURE 10. Total work time for each stage (S1-S6) in Table 2

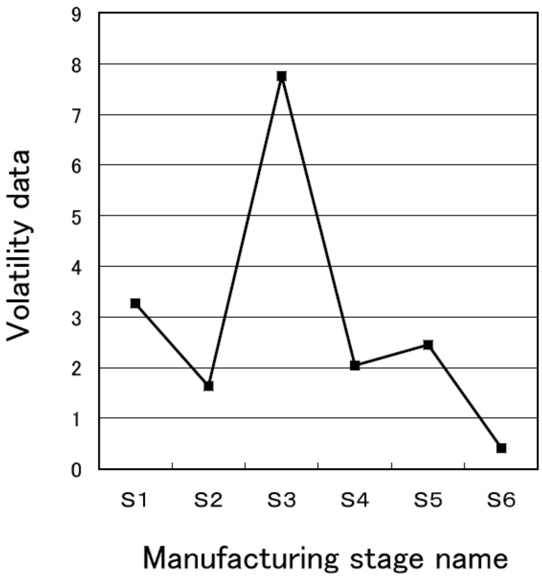

FIGURE 11. Volatility data for each stages (S1-S6) in Table 2

S3 is 199 (min), which causes a bottleneck. In Figure 10, we plot the measurement data listed in Table 2, which represents the total working time of each worker (K1-K9). In Figure 11, we plot the data contained in Table 2, which represents the volatility of the working times.

- (Test-run2): Set to synchronously process the throughput. The target time listed in Table 4 is 500 (min), and the theoretical throughput (not including the synchronization idle time) is 400 (min). Table 5 presents the volatility of each working process (S1-S6) for each worker (K1-K9).
- (Test-run3): Introduce a preprocess stage. The process throughput is performed synchronously with the reclassification of the process. As shown in Table 6, the theoretical throughput (not including the synchronization idle time) is 400 (min). Table 7 presents the volatility of each working process (S1-S6) for each worker (K1-K9). On the basis of these results, the idle time must be set to 100 (min). Moreover, the theoretical target throughput (T_s') can be obtained using the "Synchronization

Table 4. Total manufacturing time at each stages for each worker

	WS	S1	S2	S3	S4	S5	S6
K1	20	20	24	20	20	20	20
K2	20	20	20	20	20	22	20
K3	20	20	20	20	20	20	20
K4	20	25	25	20	20	20	20
K5	20	20	20	20	20	20	20
K6	20	20	20	20	20	20	20
K7	20	20	20	20	20	20	20
K8	20	27	27	22	23	20	20
K9	20	20	20	20	20	20	20
Total	180	192	196	182	183	182	180

Table 5. Volatility of Table 4

K1	0	1.33	0	0	0	0
K2	0	0	0	0	0.67	0
K3	0	0	0	0	0	0
K4	1.67	1.67	0	0	0	0
K5	0	0	0	0	0	0
K6	0	0	0	0	0	0
K7	0	0	0	0	0	0
K8	2.33	2.33	0.67	1	0	0
K9	0	0	0	0	0	0

Table 6. Total manufacturing time at each stages for each worker, K5 (*): Preprocess

	WS	S1	S2	S3	S4	S5	S6
K1	20	18	19	18	18	18	18
K2	20	18	18	18	18	18	18
K3	20	21	21	21	21	21	21
K4	16	13	11	11	13	13	13
K5	16	*	*	*	*	*	*
K6	16	18	18	18	18	18	18
K7	16	14	14	13	14	14	13
K8	20	22	22	22	22	22	22
K9	20	20	20	20	20	20	20
Total	148	144	143	141	144	144	143

Table 7. Volatility of Table 6 K5 (*): Preprocess

K1	0.67	0.33	0.67	0.67	0.67	0.67
K2	0.67	0.67	0.67	0.67	0.67	0.67
K3	0.33	0.33	0.33	0.33	0.33	0.33
K4	1	1.67	1.67	1	1	1
K5	*	*	*	*	*	*
K6	0.67	0.67	0.67	0.67	0.67	0.67
K7	0.67	0.67	1	0.67	0.67	1
K8	0.67	0.67	0.67	0.67	0.67	0.67
K9	0	0	0	0	0	0

with preprocess" method. This goal is as follows:

$$\begin{aligned} T_s &\sim 20 \times 6 \text{ (First cycle)} + 17 \times 6 \text{ (Second cycle)} \\ &\quad + 20 \times 6 \text{ (Third cycle)} + 20 \text{ (Previous process)} + 8 \text{ (Idol-time)} \\ &\sim 370 \text{ (min)} \end{aligned} \tag{25}$$

The full synchronous throughput in one stage (20 min) is

$$T_s^{'} = 3 \times 120 + 40 = 400 \text{ (min)} \tag{26}$$

Using the "Synchronization with preprocess" method, the throughput is reduced by approximately 10%. Therefore, we showed that our proposed "Synchronization with preprocess" method is realistic and can be applied in flow production systems. Below, we represent for a description of the "Synchronization with preprocess".

In Table 6, the working times of the workers K4, K7 show shorter than others. However, the working time shows around target time. Next, we manufactured one piece of equipment in three cycles. To maintain a throughput of six units/day, the production throughput must be as follows:

$$\frac{(60 \times 8 - 28)}{3} \times \frac{1}{6} \simeq 25 \text{ (min)} \tag{27}$$

where the throughput of the preprocess is set to 20 (min). In Equation (27), the value 28 represents the throughput of the preprocess plus the idle time for synchronization. Similarly, the number of processes is 8 and the total number of processes is 9 (8 plus the preprocess). The value of 60 is obtained as 20 (min) $\times$ 3 (cycles).

In Table 1, Test-run3 indicates a best value for the throughput in the three types of theoretical working time. Test-run2 is an ideal production method. However, because it is difficult for talented worker, Test-run3 is a realistic method.

The results are as follows. Here, the trend coefficient, which is the actual number of pieces of equipment/the target number of equipment, represents a factor that indicates the degree of the number of pieces of manufacturing equipment.
Test-run1: 4.4 (pieces of equipment)/6 (pieces of equipment) = 0.73,
Test-run2: 5.5 (pieces of equipment)/6 (pieces of equipment) = 0.92,
Test-run3: 5.7 (pieces of equipment)/6 (pieces of equipment) = 0.95.
Volatility data represent the average value of each Test-run.

7. **Conclusions.** Using our proposed method, we showed that the differences between the synchronous and asynchronous models are because of volatility.

Using the measured data, we derived the model equation for three types of production systems. When the synchronization method is applied to the production flow process, it achieves the best improvement of throughput. However, in practice, depending on the skills of workers, it may require a longer time. In practice, the "Synchronization with preprocess" method is the most desirable.

However, when acquiring data to assess the overall lead time on the basis of the ability of individual workers and the function of the work table, we must consider the configuration of the line to understand the constraints of production.

Acknowledgment. We thank Dr. E. Chikayama, Associate professor of Niigata University of International and Information Studies, for verifying the log-normal distribution type data.

REFERENCES

[1] M. E. Mundel, *Improving Productivity and Effectness*, Prentice-Hall, NZ, 1983.
[2] A. Neely, M. Gregorey and K. Platts, Performance measurement system design, *International Journal of Operations & Production Management*, vol.15, no.4, pp.5-16, 1995.
[3] K. Nishioka, Y. Mizutani, H. Ueno et al., Toward the integrated optimization of steel plate production process – A proposal for production control by multi-scale hierarchical modeling –, *Synthesiology*, vol.5, no.2, pp.98-112, 2012.
[4] S. Treville, R. D. Shapiro and A. Hameri, From supply chain to demand chain: The role of lead time reduction in improving demand chain performance, *Journal of Operations Management*, vol.21, no.6, pp.613-627, 1995.
[5] I. P. Tasiopoulos and B. G. Kingsman, Lead time management, *European Journal of Operational Research*, vol.14, no.4, pp.351-358, 1983.
[6] S. Hiiragi, The significance of shortening lead time from a business perspective, *MMRC*, http://merc.e.u-tokyo.ac.jp/mmrc/dp/index.html, 2012 (in Japanese).
[7] N. Ueno, M. Kawasaki, H. Okuhira and T. Kataoka, Mass customization production planning system for muli-process, *Journal of the Faculty of Management and Information Systems, Prefectural University of Hiroshima*, no.1, pp.183-192, 2009 (in Japanese).
[8] H. L. Lee, V. Padmanabhan and S. Whang, The bullwhip effect in supply chains, *Sloan Management Review*, pp.93-102, 1997.

[9] H. Kondo and K. Nisinari, Modeling stock congestion in production management, *Reports of RIAM Symposium, Mathematics and Physics in Nonlinear Waves*, no.20, pp.146-149, 2008 (in Japanese).
[10] S. J. Baderstone and V. J. Mabin, A review Goldratt's theory of constraints (TOC) – Lessons from the international literature, *Operations Research Society of New Zealand the 33rd Annual Conference*, University of Auckland, New Zealand, 1998.
[11] K. Shirai and Y. Amano, Production density diffusion equation propagation and production, *IEEJ Transactions on Electronics, Information and Systems*, vol.132-C, no.6, pp.983-990, 2012.
[12] K. Shirai and Y. Amano, A study on mathematical analysis of manufacturing lead time – Application for deadline scheduling in manufacturing system, *IEEJ Transactions on Electronics, Information and Systems*, vol.132-C, no.12, pp.1973-1981, 2012.
[13] K. Shirai, Y. Amano and S. Omatu, Process throughput analysis for manufacturing process under incomplete information based on physical approach, *International Journal of Innovative Computing, Information and Control*, 2013 (in press).
[14] P. Wilmott, *Derivatives*, John Wiley & Sons, 1998.
[15] H. Tasaki, *Thermodynamics – From a Modern Point of View (New Physics Series)*, Baifukan, Co., LTD., 2000.
[16] K. Kitahara, *Nonequilibrium Statistical Mechanics*, Iwanami, Co., LTD., 2000.