International Journal of Innovative Computing, Information and Control
Volume **10**, Number **2**, April **2014**
ICIC International ©2014 ISSN 1349-4198
pp. 601–616

NONLINEAR CHARACTERISTICS OF THE RATE OF RETURN IN THE PRODUCTION PROCESS

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Received May 2013; revised September 2013

Abstract. *We analyze a production process based on the nonlinearity of the rate of return on sales. The mathematical model leads to a van der Pol differential equation. We propose a stochastic field that is analogous to the mechanical field of physics. In the stochastic field of a production process with nonlinear characteristics, we clarify the condition that yields a stable cyclic solution using cost parameters and nonlinear characteristics parameters. We consider that overcoming the nonlinearity of production will lead to improved productivity. In a real production process, we introduce a case of throughput improvement by implementing a recombination of the process.*
Keywords: Production field, Potential energy, Van der Pol differential equation, Production density

1. **Introduction.** Several studies have addressed the problem of increasing the productivity of production processes used in the production industry [1, 2]. Moreover, in the field of production, various theories have been applied to improve and reform production processes and increase productivity.

In a previous study [3], we addressed the problem of reducing construction work and inventory in the steel industry. Specifically, we investigated the relationship between variations in the rate of construction and delivery rate. In this study, we perform analysis using the queuing model and apply log-normal distribution to model the system in the steel industry [3].

Moreover, several studies have reported approaches that lead to shorter lead times [4, 5]. From order products, lead time occurs on the work required preparation of the members for production.

Many aspects can potentially affect lead time. For example, from order products, the lead time from the start of development to the completion of a product is called the time-to-finish time, such as the work required preparation of the members for production equipments.

Moreover, several studies have focused on reducing customer lead times. In [6], the author addresses the problem of reducing the production lead time.

In [7], the authors propose a method that increases both production efficiency and production of a greater diversity of products for customer use. Their proposed approach results in shortened lead times and reduces the uncertainty in demand. Their method captures the stochastic demand of customers and produces solutions by solving a nonlinear stochastic programming problem.

In summary, several studies have considered uncertainty and proposed practical approaches to shorten the lead time. The demand is treated as a stochastic variable and apply mathematical programming. To our knowledge, previous studies have not treated lead time as a stochastic variable.

Because fluctuations in the supply chain and market demand and the changes in the production volume of suppliers are propagated to other suppliers, their effects are amplified. Therefore, because the amounts of stock are large, an increase or decrease of the suppliers' stock is modeled using differential equation. This differential equation is said as Billwhip model, represents a stock congestion [8, 9].

The theory of constraints (TOC) describes the importance of avoiding bottlenecks in production processes [10]. When using production equipment, delays in one production step are propagated to the next. Hence, the use of production equipment may lead to delays. In this study, we apply a physical approach and regard each step as a continuous step. By applying this approach, we can mathematically analyze the delay of each step and obtain methods to address it. To the best of our knowledge, previous studies have not applied physical approaches to analyze delays.

In a previous study [12], we constructed a state in which the production density of each process corresponds to the physical propagation of heat [17]. Using this approach, we showed that a diffusion equation dominates the production process.

In other words, when minimizing the potential of the production field (stochastic field), the equation, which is defined by the production density function $S_i(x,t)$ and the boundary conditions, is described using the diffusion equation with advection to move in transportation speed ρ. The boundary conditions mean a closed system in the production field. The adiabatic state in thermodynamics represents the same state [12].

To achieve the goal of a production system, we propose using a mathematical model that focuses on the selection process and adaptation mechanism of the production lead time. We model the throughput time of the production demand/production system in the production stage by using a stochastic differential equation of log-normal type, which is derived from its dynamic behavior. Using this model and the risk-neutral integral, we define and compute the evaluation equation for the compatibility condition of the production lead time. Furthermore, we apply the synchronization process and show that the throughput of the production process is reduced [13, 14].

We propose a production field for the production process (termed stochastic field in the production process) that is analogous to the mechanical field of physics [12]. The behavior of the production field is expressed as a partial differential equation describing a dynamic Hamilton-Jacobi field [18].

We also clarify the stable cycle conditions of the nonlinear characteristic parameter in the production field of the production process. To date, nonlinearity in the rate of sales return in the production process has not been reported. However, the data analyzed in this paper suggests that the rate of sales return can become nonlinear. The nonlinear element is the portion of production costs that cannot be directly attributed to sales.

Based on the rate of return on net sales, this paper mathematically analyzes nonlinearity in the rate of return on sales. The mathematical model is described by a van der Pol differential equation. Moreover, we clarify the stable cycle condition of the non-linear characteristic parameter in the production field of manufacturing process.

We propose improving productivity by overcoming the non-linearity of the production process. We then focus on synchronization of the process that leads to higher productivity. We introduce throughput improvement by implementing a recombination of processes. To the best of our knowledge, improving productivity by considering the non-linearity of the rate of return on sales has not been discussed by any previous study.

2. **Production Systems in the Manufacturing Equipment Industry.** The production methods used in production equipment are briefly covered in this paper. More information is provided in our report [13].

This system is considered to be a "Make-to-order system with version control", which enables production after orders are received from clients, resulting in "volatility" according to its delivery date and lead time. In addition, there is volatility in the lead time, depending on the content of the make-to-order products (production equipment).

In Figure 1(A), the "Customer side" refers to an ordering company and "Supplier (D)" means the target company in this paper. The product manufacturer, which is the source of the ordered production equipment presents an order that takes into account the market price. In Figure 1(B), the market development department at the customer's factory receives the order through the sale contract based on the predetermined strategy.

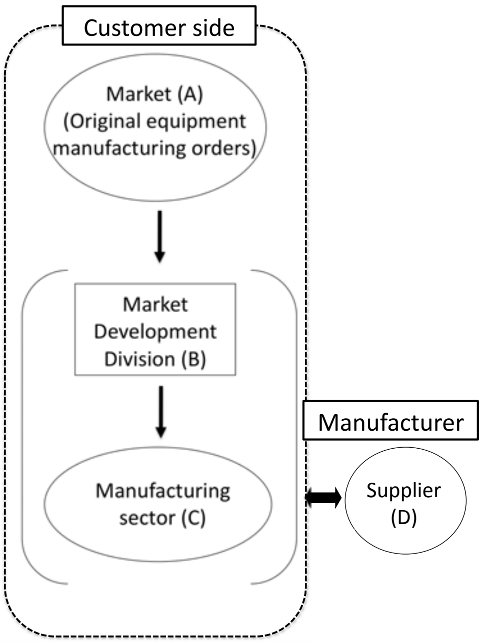

FIGURE 1. Business structure of company of research target

3. **Production Cost and Potential Energy in Production Process.** Nonproductivity generates a static state in the production field. Transition to the dynamic state, modeled by the Hamilton-Jacobi equation, requires excitation energy, which increases the free energy of the system [18].

To retain a profitable business, products must be continually input to the static field. At the same time, sustained input of order information is required. Figure 2 is an overview of the production field concept.

The number of production units at each stage of a production unit i shifts over time. To function effectively, a production process requires a minimum number of personnel. This situation constitutes a shortest path problem. Production units can be considered to be physically located in mechanical fixtures. The production dynamics enable a company to profit from its business.

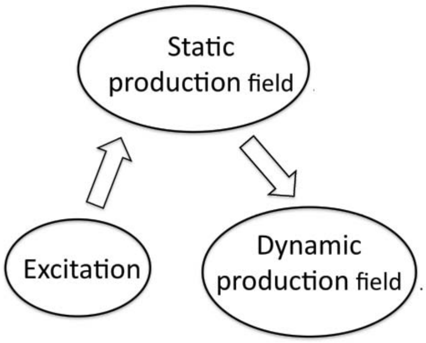

FIGURE 2. Overview of the production field concept

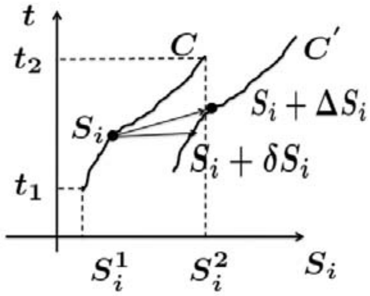

FIGURE 3. Transition from a lower-energy production process to the next process

We consider that revenues are generated by the displacement of the potential in the production field. In other words, the entropy increase contributed by the production unit is another source of revenue. This is the principle of maximum entropy [18]. Figure 3 illustrates the transition from a lower-energy production process (energy state C) to the (higher-energy) next process (energy state C').

Definition 3.1. *Production cost $S_i(t)$*

$$S_i(t) \equiv S_i^*(t) \pm \Delta S_i(t) \tag{1}$$

where the production cost $S_i^*(t)$ incorporates cost fluctuations. We now derive the model equation that constrains the dynamic behavior of the production cost.

As illustrated in Figure 4, profitability and production costs constitute the total potential of the system.

If the production field sets $\{S_i(t)\}$, $i = 1, \cdots, n$, introducing sustainable order information and exciting the system with a sustainable target allows the process to progress from a static to a dynamic production field. The free energy of the process is increased by this transition [18].

Costs are classified according to Figure 5. "Direct production cost", which relates directly to production processes, represents labor and material costs.

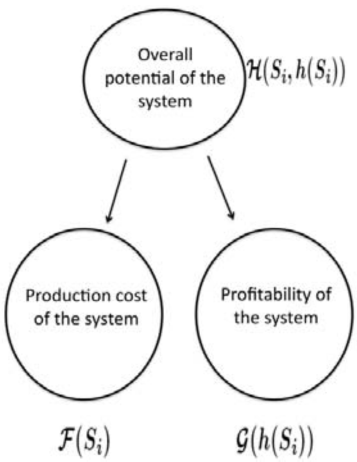

FIGURE 4. Overall potential of the system

Production costs	
Direct production costs(Man power, materials,etc)	Improvement power cost of production (Technology,etc)

FIGURE 5. Direct production cost and limitation cost of production

Definition 3.2. *The rate of return specifies the variation of the production cost; that is, the rate of return $h_{S_i}(t)$ generated by improvement expenditure is as follows:*

$$h_{S_i}(t) \cong \frac{dS_i}{dt} \tag{2}$$

Definition 3.3. *Mixing potential energy $(\mathcal{H}(S_i, h_{S_i}(t)))$*

$$\mathcal{H}(S_i, h_{S_i}(t)) \equiv \mathcal{F}(S_i) + \mathcal{G}(h_{S_i}(t)) \tag{3}$$

where $\mathcal{G}(h_{S_i}(t))$ denotes the production cost of improvement. The production cost is proportional to the rate of return. As an example, we consider the rate of return generated from technical proficiency. Because technical proficiency includes improvement power, it is hereafter referred to as "improvement power".

In terms of $\mathcal{H}(S_i, h_{S_i})$, we have

$$k_s \frac{dS_i}{dt} = \frac{\partial H(S_i, h_{S_i})}{\partial S_i} \tag{4}$$

$$k_h \frac{dh_{S_i}}{dt} = \frac{\partial \mathcal{H}(S_i, h_{S_i})}{\partial h_{S_i}} \tag{5}$$

where k_s and k_h are constants. Equation (4) describes the time variation of direct production costs, where "Direct production cost" embraces labor and material costs.

Equation (5) represents the time variation of all rate of returns. $\mathcal{H}(S_i, h_{S_i})$ is referred to as the mixing potential energy,

Definition 3.4. *Total rate of return $h_T(S_i, t)$*

$$h_T(S_i, t) \equiv \frac{\partial \mathcal{F}(S_i, h_{S_i})}{\partial S_i} + \mathcal{G}(h_{S_i}(t)) \tag{6}$$

where $\mathcal{G}(h_{S_i}(t))$ represents the cumulative rate of returns generated by improvement expenditure in Equation (7).

The total rate of return of a company in the production field is generated from both the time variation of the direct production cost and the "cumulative improvement cost for production". The time variation of production costs is assimilated into "Production cost" in the production system. "Purchase cost" comprises the purchase of parts and other items used in the production. "Production cost with variation" is paid to external production contractors.

Also included in the cost is "Transaction costs (sales volume)" as a source of rate of return. "Cumulative improvement cost for production" corresponds to "Technical proficiencies" as described above.

$$\mathcal{G}(h_{S_i}(t)) = \frac{1}{k_2} \int S_i(t) dt \tag{7}$$

Definition 3.5. *Potential energy in production field $\mathcal{F}$*

$$\mathcal{F} = \int f(S_i(t), h_{S_i}(t)) dS_i, \quad {}^{\forall}\, 0 \le t \le T \tag{8}$$

The deviation of the potential energy of production (the deviation of free energy) in the production field generates a rate of return. However, not all working costs necessarily lead to profit. Losses and disabled production costs (including inevitable revenue losses) are also included.

On the other hand, $\mathcal{H}(S_i, h_{S_i}(t))$ fluctuates with $h_T(S_i)$. Revenue is generated by deviations in $\Delta\mathcal{H}(S_i, h_{S_i}(t))$. In summary, revenue is analogous to deviations in released energy.

The above discussion provides a physical interpretation of the Hamilton canonical production field [18].

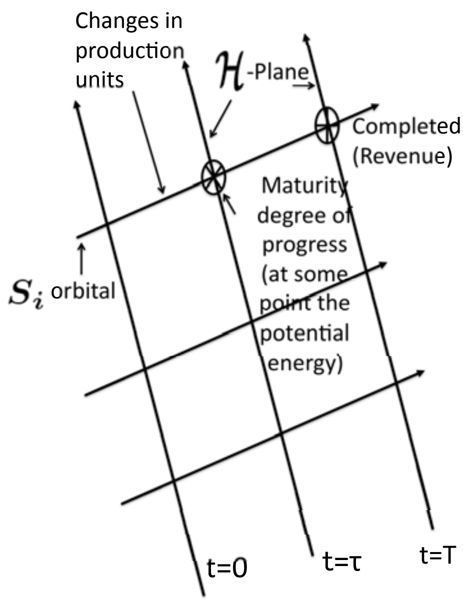

FIGURE 6. Potential energy fluctuation concept

Figure 6 shows a field $\mathcal{H}(S_i, h_{S_i}(t))$ intersecting a production space. $\mathcal{H}(S_i, h_{S_i}(t))$ on the constant surface moves next intersection during time elapsing. The Hamilton-Jacobi equation defines the temporal and spatial variation of the field. "Completed (Return)" in Figure 6 means that revenue is sourced by completing a production operation.

Here, the total rate of return is related to the potential energy of the total production cost as follows:

$$h_T(S_i) \approx \frac{d\mathcal{F}(S_i)}{dS_i} \tag{9}$$

Therefore, from Equations (9), (6) and (7), we can obtain total production cost corresponding to total rate of return as follows:

$$\frac{d\mathcal{F}(S_i)}{dS_i} \cong k_1 \frac{dS_i(t)}{dt} + \frac{1}{k_2} \int_0^t S_i(\tau) d\tau \tag{10}$$

where $\frac{dS_i(t)}{dt}$ represents the cost variation per unit time and $\frac{1}{k_2} \int_0^t S_i(\tau) d\tau$ is the cumulative cost function. The constants k_1 and k_2 are referred to as the transform coefficients of the rate of return.

Thus, we find that the total rate of return is proportional to the cumulative cost function of the target product per hour. However, if the elasticity of cost to the rate of return per unit time is positive, the process increases the rate of return. In the opposite case (decreased throughput), the process decreases the rate of return.

When the company sets a semi-fixed price for a transaction cost $N(t)$ (depending on production equipment), the rate of return $S_i(t)$ depends on the production costs and develops a nonlinear characteristic. This trend represents the structure of the rate of return in the company. Although this study assumes specific equipment, a wide variety of equipments are used in real production processes.

4. **Rate of Return and Nonlinear Characteristic of Net Sales.** Figures 7-9 display graphs in which no significant difference is apparent between cumulative revenues related to production costs and revenues related to production throughput.

Figures 7-9 plot the rate of return on net sales of specific control equipment produced by some domestic enterprises from 1996 through 1998. The rate of return on sales gives rise to the nonlinear characteristics.

The dashed line in the figures is the fitted curve representing the relationship between the rate of return on sales and sales volume fee. In the data, the return rate plummeted from 0.3 at a sales fee of 480 to 0.15 at a sales fee of 440 (see Figure 7). This sharp drop represents the relationship in Equation (17).

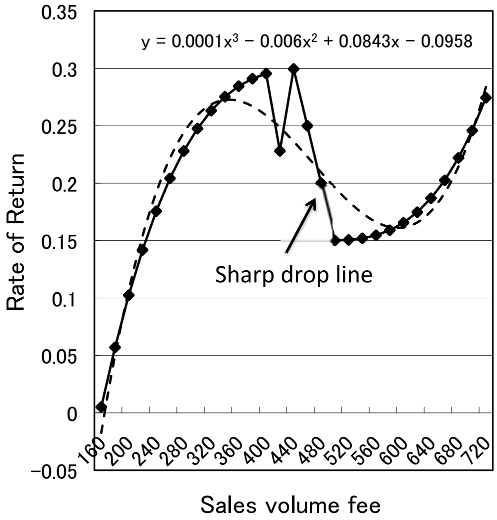

FIGURE 7. Rate of return on sales volume 1

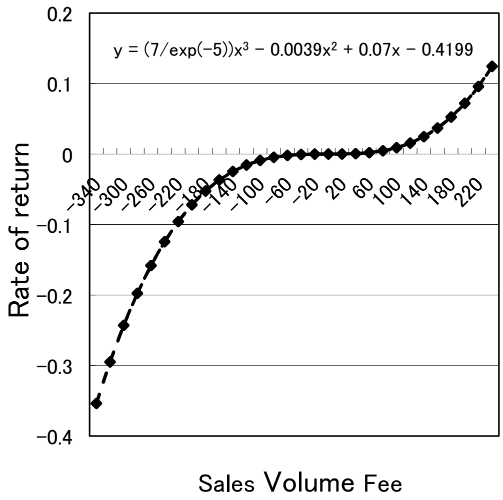

FIGURE 8. Rate of return on sales volume 2

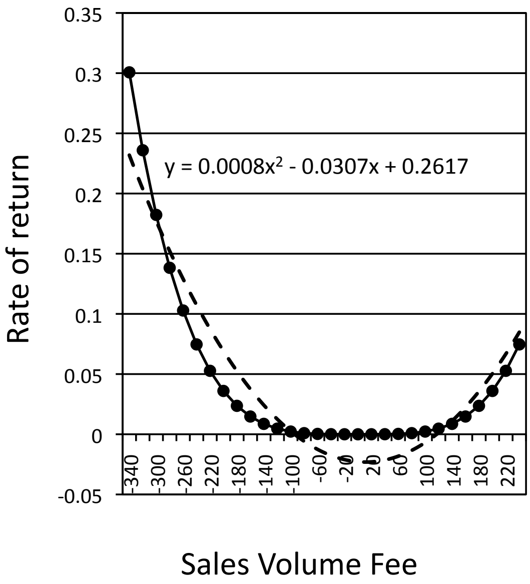

FIGURE 9. Rate of return on sales volume 3

The resulting straight line appears in the vicinity of the phase transition and is equivalent to the oscillation point of the reference line in elements displaying nonlinear characteristics (such as the Esaki diode) [11].

$$h_s(S) = F(S) + \xi(h_{s_0}) \tag{11}$$

where $F(S)$ represents the basic characteristics of the return rate, and $\xi(h_{s_0})$ is a neighborhood of local nonlinearity around h_{s_0}. The following mathematical model is derived from the data plotted in Figures 7-9 [11].

$$a\frac{dh_s}{dt} + bh_s + S = S_E \tag{12}$$

$$h_s = h_{s_1} + h_{s_2} \tag{13}$$

$$h_{s_2} = \tilde{F}(S') \tag{14}$$

$$S' = c\int h_{s_1} dt \tag{15}$$

$$\tilde{F}(S') = F(S) + \xi(h_{S_0}) \tag{16}$$

$$S_E - bh_{s_0} = S' \tag{17}$$

where a, b, and c are cost coefficients, h_s is the rate of return and h_{s_1} is the rate of return contributing to the sales volume. h_{s_2} is a nonlinear characteristic of the rate of return (introduced by costs that cannot contribute directly to sales and that lead to production delays), and (h_{s_0}, S_0) is the median of the nonlinear characteristic.

Physically, Equation (12) represents the temporal variation of the rate of return h_s; that is, the relationship between the deviation of the rate of return and the sales or rate of return. Although sales are essentially proportional to production costs, not all of the production cost can be invested in sales.

Equation (13) is the sum of the rate of return and the nonlinear element. In other words, it embodies the cost of production and nonproduction costs that make no contribution to sales.

From Equations (12)-(17), the sales S dynamics are modeled as

$$a\frac{d(h_{s_1} + h_{s_2})}{dt} + b(h_{s_1} + h_{s_2}) + S' = S_E \tag{18}$$

From Equations (14) and (15), we obtain as follows:

$$h_{s_1} = \frac{1}{c}\frac{dS'}{dt} \tag{19}$$

From Equations (16) and (17), we obtain as follows:

$$\frac{d\tilde{F}(S)}{dt} = \frac{d\tilde{F}(S)}{dS'} \cdot \frac{dS'}{dt} \tag{20}$$

$$a\Big(\frac{1}{c}\frac{d^2S'}{dt^2}\Big) + \Big(a\frac{d\tilde{F}(S)}{dS'} + \frac{b}{c}\Big)\frac{dS'}{dt} + \tilde{F}(S) + S' = S_E \tag{21}$$

Then, it is derived in Equation (21) as follows:

$$S' = S_0 + S \tag{22}$$

$$\tilde{F}(S) = F(S') + \xi(h_{s_0}) = -PS + qS^3 + \xi(h_{s_0}) \tag{23}$$

We substitute Equation (23) into Equation (21), then we obtain as follows:

$$\frac{a}{c}\frac{d^2S}{dt^2} + a\Big\{(-P + 3qS^2) + \frac{b}{c}\Big\}\frac{dS}{dt} + b\Big\{-PS + qS^3 + \xi(h_{s_0})\Big\} + S_0 + S = S_E \tag{24}$$

At this time, using Equation (17), we can obtain as follows:

$$\frac{a}{c}\frac{d^2S}{dt^2} + a\Big\{(-P + 3qS^2) + \frac{b}{c}\Big\}\frac{dS}{dt} + b\Big\{-PS + qS^3 + \xi(h_{s_0})\Big\} + S_0 + S = bh_{s_0} + S_0 \quad (25)$$

Therefore,

$$\frac{a}{c}\frac{d^2S}{dt^2} + a\Big\{(-P + 3qS^2) + \frac{b}{c}\Big\}\frac{dS}{dt} + b\Big\{-PS + qS^3 + \xi(h_{s_0}) - h_{s_0} + \frac{1}{b}S\Big\} = 0 \quad (26)$$

Assumption 1. *The nonlinear characteristic $\xi(h_{s_0})$ equals the midpoint of the rate of return.*

$$\xi(h_{s_0}) - h_{s_0} \approx 0 \quad (27)$$

$$\frac{a}{c}\frac{d^2S}{dt^2} + \left[3aqS^2 - \left(aP - \frac{b}{c}\right)\right]\frac{dS}{dt} + b\left[-PS + qS^3 + \frac{1}{b}S\right] = 0 \quad (28)$$

Then, from Equation (28), we obtain as follows:

$$\frac{d^2S}{dt^2} + \frac{c}{a}\left[\left(3aqS^2 - \left(aP - \frac{b}{c}\right)\right)\right]\frac{dS}{dt} + bqS^3 + (1 - bP)S = 0 \quad (29)$$

Therefore,

$$\frac{d^2S}{dt^2} + [\alpha S^2 - \beta]\frac{dS}{dt} + \hat{q}S^3 + \hat{P}S = 0 \quad (30)$$

Here, α and $\hat{q}$ satisfy as follows:

$$\alpha = 3cq, \quad \beta = \left[cP - \frac{b}{a}\right], \quad \hat{q} = bq, \quad \hat{P} = 1 - bP \quad (31)$$

Then, we analyze the condition in order to satisfy Equations (30) and (31).

First, in case of $\alpha > 0$ and $\beta > 0$, we obtain as follows:

1. if $f(S) = \alpha S^2 - \beta$, $g(S) = \hat{q}S^3 + \hat{P}S$, then,

$$f(S) = f(-S), \quad g(S) = -g(S)$$

   Next, if $\hat{q} > 0$ and $\hat{P} > 0$, when $S \neq 0$, $Sg(S) > 0$ and $f(0) < 0$.
2. $f(S)$ satisfies Lipschitz condition clearly, is continuos for all of S.
3.

$$F(S) = \int_0^S f(S)dS = \int_0^S (\alpha S^2 - \beta)dS = \frac{\alpha}{3}S^3 - \beta S$$

   Then, $F(S) \to \pm\infty$ is for $S \to \pm\infty$.
4.

$$F(S) = \frac{\alpha}{3}S^3 - \beta S = S\left(\frac{\alpha}{3}S^2 - \beta\right)$$

   Because $F(S)$ is a cubic function (see Figure 10), it has a single root below $F(S) = 0$ and $S = \sqrt{\frac{3\beta}{\alpha}} > 0$. Moreover, $F(S)$ is a monotonically increasing function.

Thus, $F(S)$ satisfies Lienard's theorem [11]. When nonlinearity appears in the relationship between sales and the rate of return in the production process random field (Figures 7-9), the system has a periodic solution with a unique stable trajectory. The conditions under which this occurs are $\alpha > 0$, $\beta > 0$, $\hat{P} > 0$, $\hat{q} > 0$ under above conditions (1)-(4). Thus, we obtain

$$\frac{bc}{a} < P < \frac{1}{b} \quad (32)$$

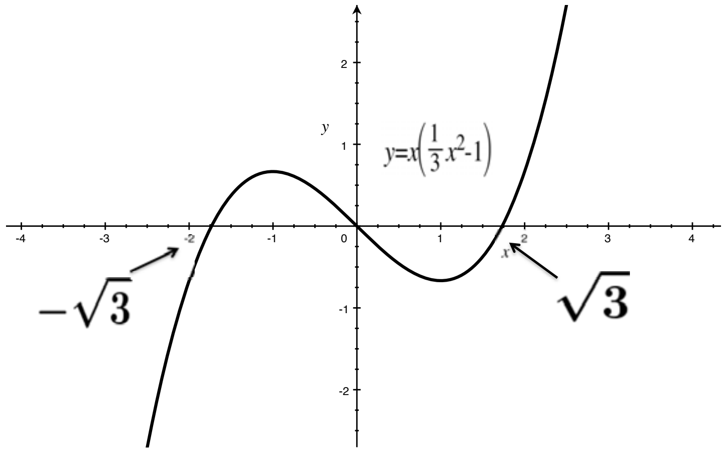

FIGURE 10. Example of three degree function

On the other hand, $b = 0$ in Equation (12); that is, when no losses occur in the production process random field (energy consumption is zero), we have

$$\begin{aligned} a\frac{dh_s}{dt} + S &= S_E \\ S &= c\int h_{s_1}dt \\ h_s &= h_{s_1} + h_{s_2} \\ h_{s_2} &= F(S) \end{aligned}$$

From this, we obtain the following equation from the same calculation.

$$\frac{d^2S}{dt^2} + \frac{c}{a}\Big[3aqS^2 - aP\Big]\frac{dS}{dt}S = 0 \tag{33}$$

That is,

$$\frac{d^2S}{dt^2} - \Big[aP - 3aqS^2\Big]\frac{dS}{dt} + S = 0 \tag{34}$$

$$\frac{d^2S}{dt^2} - Pc\Big[1 - 3\frac{q}{P}S^2\Big]\frac{dS}{dt} + S = 0 \tag{35}$$

Equation (35) is the van der Pol equation.

As mentioned above, a stable periodic solution can be obtained if the cost parameters and the parameter in the nonlinear characteristic satisfy Equation (32) in the random field of production processes, such as those plotted in Figures 7-9.

A numerical simulation of the system with $S_i(0) = 0.5$ and $\dot{S}_i(0) = 0$ is plotted in Figure 11.

The amplitude (energy) initially increases, after which the sine wave repeats at a constant amplitude; that is, the energy balance is steady over time, and the production industry operates smoothly.

If we now set $S_i(0) = 0.5$ and $\dot{S}_i(0) = 0$, the situation shown in Figure 12 emerges, in which energy has converged.

Figure 13 plots the numerical output of the system for $S_i(0) = 0.5$ and $\dot{S}_i(0) = 0$.

At a large ϵ ($\epsilon = 10$), a sawtooth waveform develops with sharp teeth along one edge. In this situation, the manufacturing industry maintains a low production throughput.

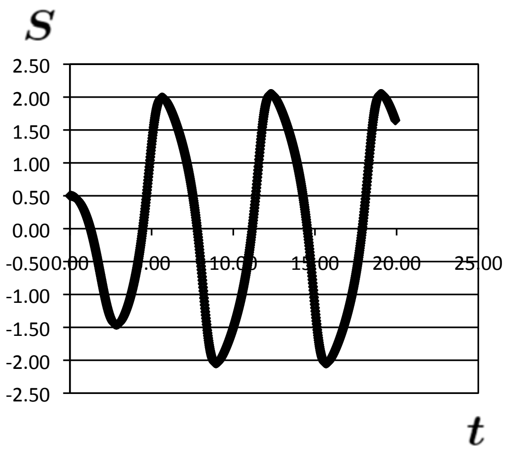

FIGURE 11. van der Pol equation ($\epsilon = 1$)

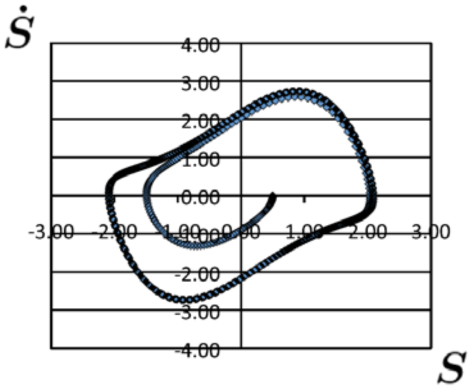

FIGURE 12. Relationship between S_i and $\dot{S}_i$ ($\epsilon = 1$)

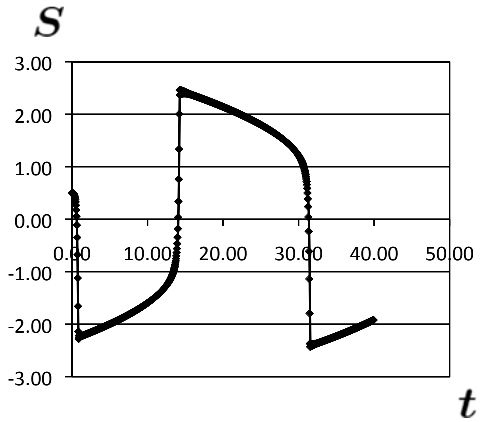

FIGURE 13. van der Pol equation ($\epsilon = 10$)

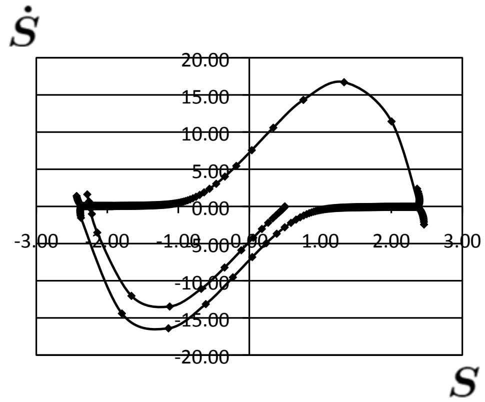

FIGURE 14. Relationship between S_i and $\dot{S}_i$ ($\epsilon = 10$)

Production costs comprise a large portion of expenditure and are responsible for the section of slow change in the waveform. We refer to this process as a "Bottleneck process".

In processes other than bottleneck, the production costs consume a relatively small portion of the expenditure. In other words, the production throughput is relatively high.

Figure 14 plots the numerical solution with $S_i(0) = 0.5$ and $\dot{S}_i(0) = 0$. The slow periodic motion manifests as a sawtooth waveform with sharp teeth along one edge.

Therefore, the active elements in the production are synchronized to minimize the production losses. In other words, synchronization is a production strategy by which the overall production cost can be implemented while minimizing the average local potential energy.

This strategy is referred to as "Bottleneck synchronization" in TOC, and the "Principle of minimizing losses" in Physics [10, 16].

5. **Production Process Improvement Example that Overcomes the Non-Linearity.** After we observed the nonlinear characteristics in the production process, we focused on an attempt to improve throughput [13]. At present, we have maintained a synchronized process. Using asynchronous logistics phenomenon and supply chain delays, we

present a throughput improvement example, in which a production flow process is used for throughput improvement.

Here we investigate improved and standard process flows using a control device as an example. As a result, we found that according throughput improvement post-process priority is appropriate. Using a buffer of the previous process to overcome bottlenecks in the post process, the previous process can synchronize the post process, leading to significantly improved lead time.

Figure 15 illustrates the concept of process synchronization. Here X_{PR} represents the previous process, X_P represents the pre-work start date of the post process, and X_M represents the start date of the post process.

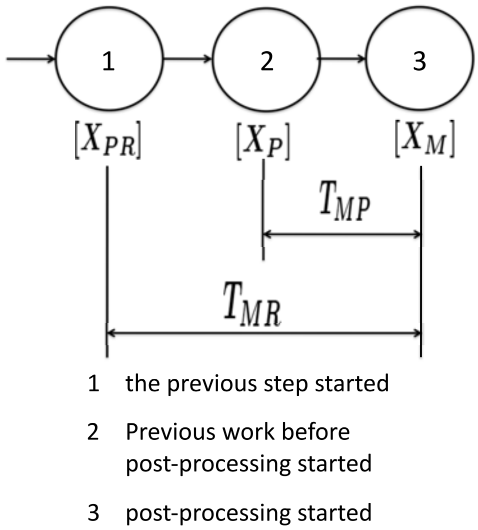

FIGURE 15. Conceptual diagram of the production process synchronization

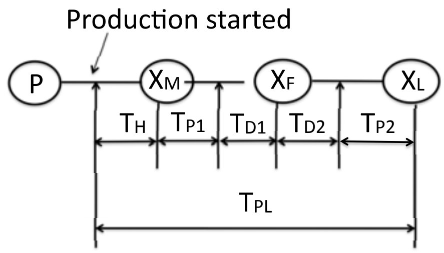

FIGURE 16. Production lead time in entire process

If we set the required production number $S(X_M)$ (i.e., required production number in a post process) to a synchronization point in time X_M, there is at least the following relationship between production numbers $S_P(X_{MP})$ among $[T_{MP}]$ and production numbers $S_R(X_{PR})$ among $[T_{MR}]$:

$$S_M(X_M) \leq S_P(X_{MP}) + S_R(X_{PR}) \tag{36}$$

where each symbol is as follows.

$$S_P(X_M) \equiv k_P \cdot [T_{MP}] \cdot n_P \tag{37}$$

$$S_R(X_{PR}) \equiv k_R \cdot [T_{MR}] \cdot n_R \tag{38}$$

Here n_P and n_R are the number of working people, k_P and k_R represent the process throughput variable (i.e., number of productions/all working people), and $[T_{MP}]$ and $[T_{MR}]$ represent the lead times of each period.

$$[T_{MP}] \equiv P_P[X_{MP}] > \bar{X}_P \cdot |X_M - X_P| \tag{39}$$

$$[T_{MR}] \equiv P_R[X_{PR}] > \bar{X}_R \cdot |X_M - X_{PR}| \tag{40}$$

where when $\bar{X}_P > 0$, $\bar{X}_P$ is integer and when $\bar{X}_R > 0$, $\bar{X}_R$ is integer.

$P_P[X_{MP} > \bar{X}_P]$ and $P_R[X_{PR} > \bar{X}_R]$ are as follows:

$$P_P[X_{MP} > \bar{X}_P] = \Phi_P[\bar{X}_P/\sigma_{MP}] \quad (41)$$

$$P_R[X_{PR} > \bar{X}_R] = \Phi_R[\bar{X}_R/\sigma_{PR}] \quad (42)$$

where $\Phi_P[\bullet]$ and $\Phi_R[\bullet]$ represent standard normal distribution function respectively.

Thus, the following can be established.

$$S_M \leq S_R + S_P, \quad {}^{\forall}\, S_R > S_P \quad (43)$$

Equation (50) provides the relationship model of lead time and actual production manpower (input personnel). The lead time model is constructed from the model shown in Figure 16. We obtain several concepts from this model, i.e., the relationship between lead time and start date, the relationship between lead time and production manpower, and the lead time reduction equation. The model enables the consideration of the production flow.

Ideally, the relationship between production lead times and production start date in real companies is defined quantitatively. In particular, we select typical production equipment with different specifications for production and measure the final inspection time from start time to production completion. For any unforeseen situation, using statistical data, we can determine specific numerical targets.

We focus on the lead times of off-premise and on-premise production. In Figure 16, T_{PL} represents the production lead time, T_{P1} represents the production lead time for off-premise production (stochastic variable including deviation), T_{P2} represents the production lead time for on-premise production (stochastic variable including deviation), T_{D1} represents the residence time (idle time) of on-premise production, and T_H represents a previous process (harness processing). Thus, the production lead time can be obtained as follows.

$$T_{PL} = (T_{P1} + T_{P2}) + (T_{D1} + T_{D2}) + T_H \quad (44)$$

Here the production lead time is obtained from X_P (starting date) until X_E (production completion date) and is described as follows.

$$T_{PL} = |X_L - X_P| \quad (45)$$

If $P[T_{LM} > T_{PL}]$ provides a deviation of $|X_L - X_M|$, the evaluation of T_{DP}, which provides the production lead time of an actual process, is described as follows.

$$T_{DP} \leq T_{LM} - (T_{D1} + T_{D2}), \quad {}^{\forall}\, T_{LM} = |X_L - X_M| \quad (46)$$

$$= P[T_{LM} > T_{PL}] \cdot |X_L - X_M| - (T_{D1} + T_{D2}) \quad (47)$$

Here we refer to $P[T_M > T_{PL}]$ as an incompatibility factor versus $|X_L - X_M|$, where M is any positive integer.

Example 5.1. *If the risk rate is 5%,* $|X_L - X_M| = 18$ *(date) and* $(T_{D1} + T_{D2}) = 5$ *(days); thereafter,* T_{DP} *can be obtained as follows.*

$$T_{DP} \leq 0.95 \times 18 - 5 = 12 \quad (48)$$

From Equation (48), a post process must be completed within 12 days.

From the above description, we can evaluate the standard lead time in a post process in advance. Therefore, if the standard lead time is measured as $[T_{LM}]_{nom.}(h)$, the production lead times is as follows.

$$[T_{PL}]_{nom.} \geq \frac{[T_{LM}]_{nom.}(h) + T_H(h)}{8n(people)} \quad (49)$$

Thus, we can conduct a production process within the standard process time. We rewrite Equation (49) for $n(people)$. Then, we can obtain the production lead time as follows.

$$n \geq \frac{[T_{LM}]_{nom.}(h) + T_H(h)}{8 \cdot [T_{PL}]_{nom.}} \tag{50}$$

Figure 17 can obtain from Equation (49). Figure 18 illustrates the standard production flow for equipment and represents a real production flow diagram rather than the lead time concept shown in Figure 16. Figure 20 illustrates the measurement lead time for a real production number. From the above description, if n_P and n_R are fixed, we have no choice but to alter the production rate to satisfy the synchronization condition. Considering risk in lead times, it is best to employ process flattening and process coupling.

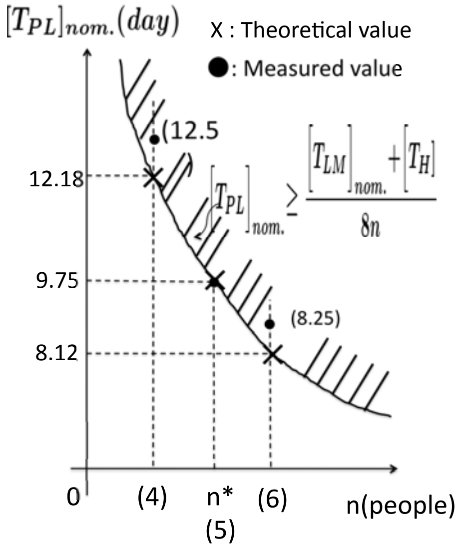

FIGURE 17. Relationship between lead time and work people

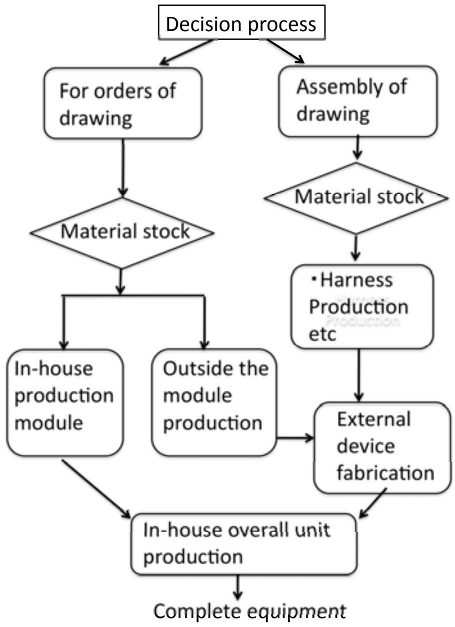

FIGURE 18. Standard equipment fabrication flow

To control the production capacity variable, we must deploy fair and flexible manpower planning and measure the lead time of production equipment. Figure 18 shows a standard production flow, and Figure 19 illustrates an improved flow obtained by flattening a cable manufacturing process. By incorporating a cable manufacturing process as a pre-process, we were able to obtain an improved process. Figure 20 shows the measurement results of production lead time from data obtained for a produced device. Here after receiving an order to manufacture equipment and confirming parts distribution, we can determine the start date by considering the delivery date, as is shown in Figure 20.

Then, Figure 20 provides the actual measurement data, which is the lead time of each process and time until final inspection is completed from the start date of production.

The production lead times are obtained by (measurement lead time)/(standard lead time).

Here the average production lead time is 1.0275 and the standard deviation is 0.051. From these results, the production lead times are relatively stable; however, a minor difference occurs in production lead times due to production equipment specifications.

Thus, we calculate the reduction rate of lead time to obtain (improved production flow)/(standard production flow) = 0.826 in the improved production flow 1, and (improved production flow)/(standard production flow) = 0.7239 in the improved production flow 2.

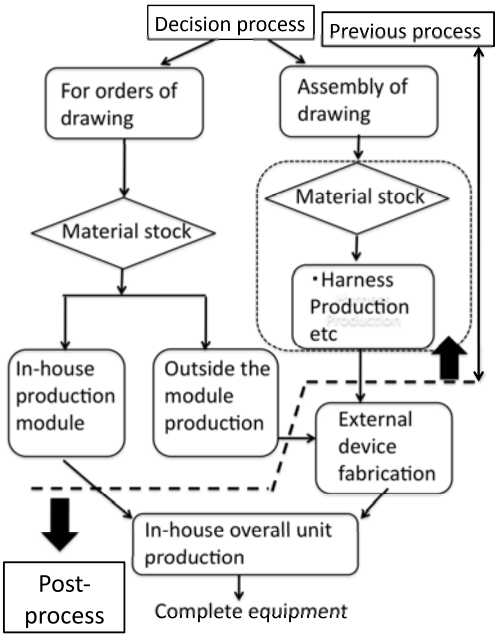

FIGURE 19. Improved equipment fabrication flow

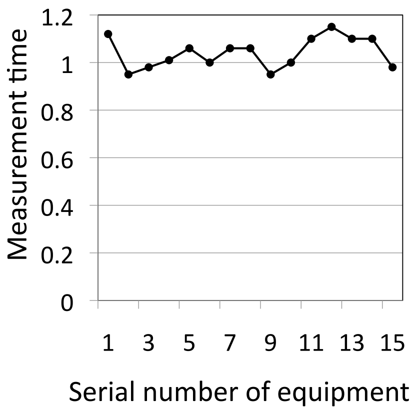

FIGURE 20. Production lead time variability

Therefore, the reduction rate of lead time is improved by approximately 13% in the improved production flow 1 and is improved by approximately 20% in the improved production flow 2. Here we define a throughput coefficient based on a standard production flow as follows.

Definition 5.1. *Definition throughput coefficient based on a standard production flow*

$$\eta \equiv \frac{\textit{[Number of production man – power]} \times \textit{[Number of real working time]}}{\textit{[Production risk rate]} \times \textit{[Reduction rate of lead time]}} \times \frac{1}{\textit{[Real working time of lead time]}} \tag{51}$$

If the numerator is constant, i.e., [production risk rate] = 1 and [real lead time] = constant, $\eta \cong 1.21$ (21% increase) in the improved production flow 1 and $\eta \cong 1.35$ (35% increase) in the improved production flow 2. From the above description, by using a previous process as a buffer in a post process, we can realize synchronization between a previous process and post process. In other words, we have realized a post process with priority higher than the previous process.

6. **Conclusions.** In a production flow process, the production cost is found by linking sales costs.

The nonlinear elements appearing in the system are considered to not contribute directly to sales. If all production processes can be completed within regular operating hours, production costs are invested completely in sales.

From a theoretical analysis, we have proposed a nonlinear mathematical model for production costs that is described by the van der Pol equation.

In addition, we have provided an improved production example that realized synchronization of production processes by considering the nonlinear characteristics of production processes. This is a significant contribution for an equipment manufacturer to realize throughput reduction.

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