PROFIT AND LOSS ANALYSIS ON A PRODUCTION BUSINESS USING LEAD TIME FUNCTION

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ABSTRACT. *We propose a profit and loss analysis on the outlet side of a production flow processes using a lead time function. This is the ongoing study, which analyzes a production process by using a lead time function. With respect to a production business, we need to secure operating revenue as a production company. To analyze the profit and loss on a production business, we introduce a system for the evaluation of revenue under the conditions of borrowing and capital repayments. We use the actual data obtained from a production flow process for evaluation of the break even point. With regard to a value after a repayment, whether the value is changed or not in the case where a guaranty by a company president is required is reported. In addition, how a value of manufacturing equipment (remaining value) changes after a repayment of a loan relative to a repayment period is reported. Finally, a degree (sensitivity) of influence of parameters, an initial plan money amount, a repaid money amount and a repayment period, on a remaining value and a result of risk analysis is also reported.*
Keywords: break even point, lead time function, log-normal distribution, Black-Scholes equation, stochastic differential equation

1. **Introduction.** Based on mathematical and physical understandings of production engineering, we are conducting research aimed at establishing an academic area called mathematical production engineering. As our business size is a small-to-medium-sized enterprise, human intervention constitutes a significant part of the production process, and revenue can sometimes be greatly affected by human behavior. Therefore, when considering human intervention from outside companies, a deep analysis of the production process and human collaboration is necessary to understand the potential negative effects of such intervention.

With respect to mathematical modeling of deterministic systems, a physical model of the production process was constructed using a one-dimensional diffusion equation in 2012[1]. However, the many concerns that occur in the supply chain are major problems facing production efficiency and business profitability. A stochastic partial bilinear differential equation with time delay was derived for outlet processes. The supply chain was modeled by considering as time delay[2]. With respect to the analysis of production processes in stochastic systems based on financial engineering, we have proposed that a production throughput rate can be estimated utilizing a Kalman filter based on a stochastic differential equation[3]. We have also proposed a stochastic differential equation (SDE) for the mathematical model describing production processes from the input of materials to the end. We utilized a risk-neutral principal in stochastic calculus based on the SDE[4].

With respect to the analysis of production processes based on physics, we have clarified that phenomena such as power−law distributions, self-similarity, phase transitions, and on−off intermittency can occur in production processes[5, 6, 7, 8, 9]. On the other hand, there is the famous theory of constraints (TOC) that describes the importance of avoiding bottlenecks in production processes[10]. Small fluctuations in an upstream subsystem appear as large fluctuations in the downstream (the so-called bullwhip effect)[13]. The bullwhip effect generates a large gap between the demand forecasts of the market and suppliers. Large fluctuations can be suppressed by the following mechanisms.

(1) Reducing the lead time, improving the throughput, and synchronizing the production process by the TOC.
(2) Sharing the demand information and performing mathematical evaluations.
(3) Analyzing the reduction and fluctuating demands of the subsystem (using nonlinear vibration theory).
(4) Basing the inventory management approach on stochastic demand.

When using manufacturing equipment, delays in one production step are propagated to the next. Hence, the use of manufacturing equipment itself may lead to delays. The improvement of production processes was presented that the" Synchronization with preprocess" method was the most desirable in practice using the actual data in production flow process based on the cash flow model by using the SDE of log-normal type[11]. In essence, we have proposed the best way, which is a synchronous method using the Vasicek model for mathematical finance[12]. Then, the supply chain theme, which was a time delay in the production processes, was proposed for the throughput improvement based on a stochastic differential equation of log−normal type[13].

In this study, to determine the break even point, we first construct a model for the guarantee expenses (GE), which is the present value of the total amount in the case of guaranteed repayment for all times. We also introduce an asset value of the investor, which is a sum of the present value of the remaining value (RV) that results from the subtraction of the payments from the cash flow. A break-even point refers to the point where GE and RV are equal. We validate the break-even point analysis and revenue evaluation based on the actual data obtained from a production flow process.

We also report that, in general, profit can be improved in a case where strategy to lead to an excessive production or an excessive order entry state is taken rather than a case where production is made to match the average order entry. In addition, because, also in the case of the manufacturing business that is the subject of the present research, a rate of return is distributed log-normally, a cash flow of a target company proportional to a rate of return will be also distributed log-normally, naturally. With regard to equipment manufacturing, a small-to-midsize firm is required company president's guarantee for a borrowing inevitably. Therefore, whether a value of manufacturing equipment after a repayment of a loan varies relative to a repayment period or not is reported.

Also, how a value of manufacturing equipment after a repayment of a loan (hereinafter, referred to as a remaining value) varies relative to a repayment period is reported. Finally, a degree of influence (sensitivity) of parameters of an initial plan money amount, a repaid money amount and a repayment period on a remaining value, and also a risk analysis result is reported. To the best of our knowledge, this study represents the first use of the option pricing theory for a production evaluation method.

2. **Production systems in the production equipment industry.** The production methods used in equipment are briefly covered in this paper (refer to Figure 1). Please see our research[5]. This system is considered to be a" Make-to-order system with version control," which enables manufacturing after orders are received from clients, resulting in" volatility" according to its
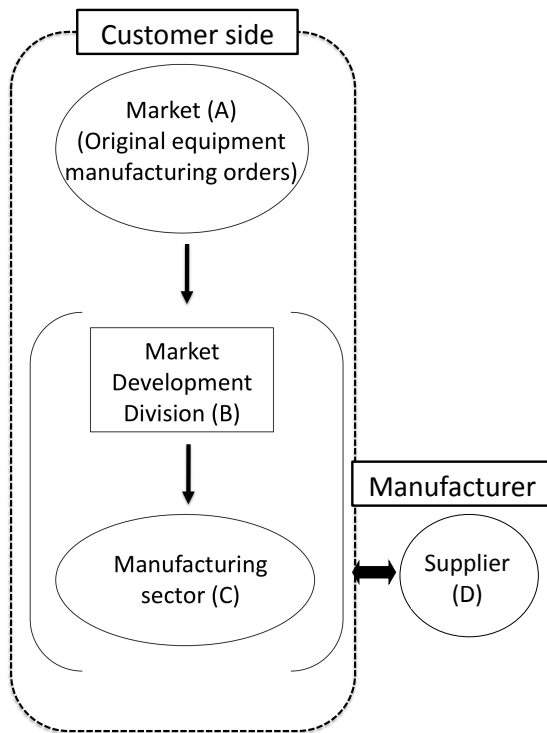
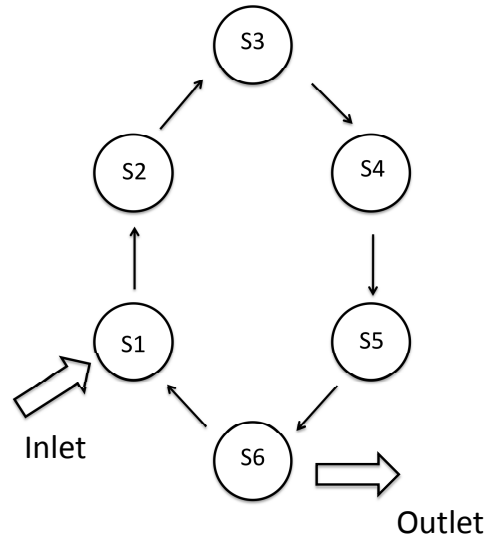FIGURE 1. Business structure of company of research target

FIGURE 2. Production flow process

delivery date and lead time. In addition, there is volatility in the lead time, depending on the content of the make-to-order products (production equipment).

2.1. **Production flow process.** A manufacturing process that is termed as a production flow process is shown in Figure 2. The production flow process, which manufacture low volumes of a wide variety of products, are produced through several stages in the production process. In Figure 2, the processes consists of six stages. In each step $S1-S6$ of the manufacturing process, materials are being produced.

The direction of the arrows represents the direction of the production flow. production materials are supplied through the inlet and the end-product is shipped from the outlet[11].

2.2. **General production framework in manufacturing business.** Generally, the disclosed information of each other between the production side of the business management side is required to realize the steadily sales volume increasing in the manufacturing industry in Figure 3.

The management side strengthens the production capacity depending on demand state of the business and provides a certain cost for facilities management. Because the business sales is determined by the production capacity and product value.

With respect to revenue, the minimum amount of stock is best. Therefore, to improve the overall revenue based on observable amount of stock must be required. Of course, we implement during the management/production department for the information sharing about an inventory.
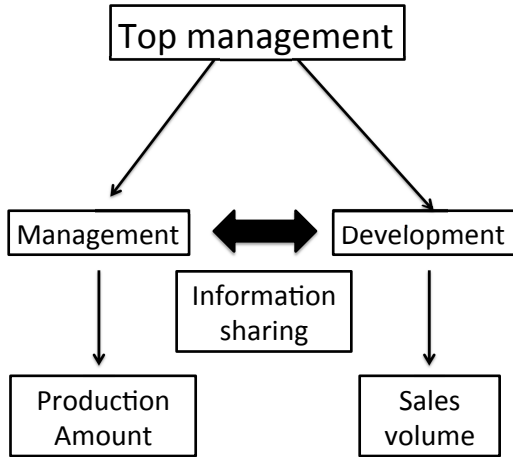
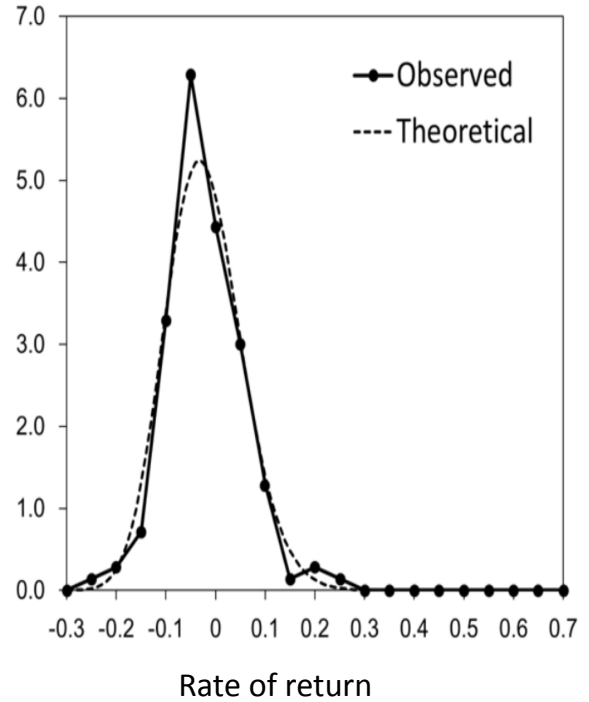FIGURE 3. Information sharing between a management div. and a development div.

FIGURE 4. Probability density function of rate-of-return deviation: actual data (solid line) and data based on theoretical equation (dotted line)

2.3. **Rate of return.** For a small-to-midsize firm, it is of the upmost importance not to cause default in a cash flow, and it is necessary for business continuity. As is the case with rate-of-return deviation described in the previous half, we also analyzed a return acquisition rate defined by Eqn. (2.1). The result is shown in Figure 4.

We place the net sales to X_t and the production capacity to C_t. Then we are able to obtain as follows[5]: From the data of monthly rate of return (RoR) observed, its probability density function was calculated (Figure 4). As a result, it was found that the probability density function conforms to log-normal distribution (PDF, Figure 4, Theoretical).

Theoretical curve was calculated using EasyFit software (http://www.mathwave.com/), and as a result of Kolmogorov and Smirnov test, the observed values conformed to a log-normal type probability density function. Because, in the goodness-of-fit test of Kolmogorov-Smirnov, a null hypothesis that it is " log-normal " was not rejected with rejection rate 0.2, this data conforms to " log-normal " distribution. $P-value$ was 0.588. The parameters of a theoretical curve were: μ_p =- 0.134 (average), σ_p = 0.0873 (standard deviation), γ_p = -0.900. The theoretical curve is given by the following equation.

$$f(x) = \frac{1}{\sqrt{2\pi}(x-\gamma_p)\sigma_p} \times \exp\left\{-\frac{1}{2}\left(\frac{(\ln x - \gamma_p)-\mu}{\sigma_p}\right)^2\right\} \tag{2.1}$$
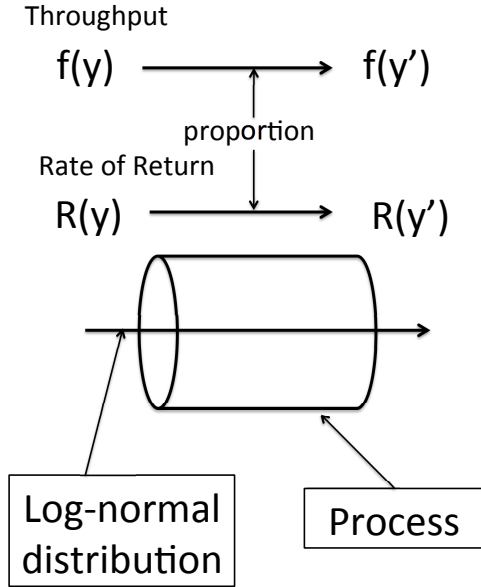
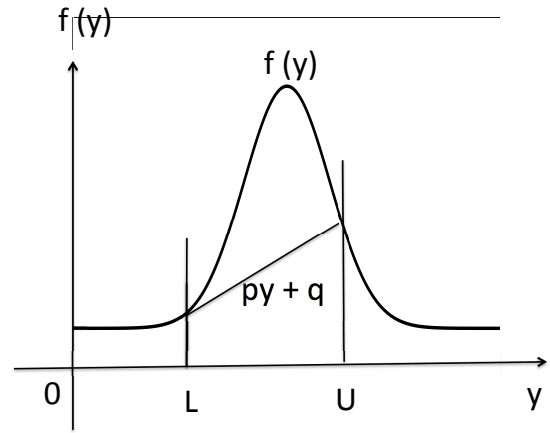FIGURE 5. Throughput fluctuation in a process distribution amount

FIGURE 6. Lead time function f(y) and Loss Function $py+q$

3. **Description of cash flow.** In a small-to-midsize firm, because it does not have ample working capital for the company, in order to continue the company operation by any means, it needs to raise working capital from financial institutions. Let this be called a cash flow. In essence, a lead time is at least proportional to a manufacturing cost.

3.1. **The introduction of lead-time function.** Figure 5 shows that a throughput is proportional to a rate of return in production processes. Then, we introduce the lead-time function so that we can analyze a production process[14]. The lead time of production equipment is proportional to the RoR. Therefore, we determined that the lead time PDF was also the same PDF of RoR. Thus, the lead-time function $f(y)$ is assumed as a log-normal probability density function so that we can calculate the lead time using a continuous expected value calculation as shown in Figure 6.

Assumption 3.1. *Lead time function of a probability density function with log-normal type.*

$$f(y) \equiv \frac{1}{\sqrt{2\pi}\sigma(y/y_0)} \exp\left\{-\frac{(\ln(y/y_0)-\mu)^2}{2\sigma^2}\right\} \tag{3.1}$$

where, μ is an average value, σ is a volatility and y_0 is an initial lead time.

Now, let C_0 as a fixed cost, and we calculate a continuous expected loss value F as a cash-in flow.

$$F = \int_{-\infty}^{\infty} f(y)B(y)dy + C_0 = \int_{-\infty}^{L} B(y)f(y)dy + \int_{L}^{U} B(y)f(y)dy$$
$$+ \int_{U}^{\infty} B(y)f(y)dy + C_0 \tag{3.2}$$

where,

$$B(y) = py+q, \quad p \geq 0 \tag{3.3}$$

where q is a constant parameter. L is a minimal lead time. L is a minimal lead time, $U = kL$ and $k (> 1)$ is a constant parameter. p is as diminishing increasing function, for example, believes that the following function. $p = \alpha \sqrt{y - kU}$, α is a constant value. U is a maximum lead time.

When $y < L$, production activities are not running. When $y > U$, the quantity ordered exceeds the physical limits of the production. Therefore, we must reduce the demand, and the problem becomes an analysis of $L \leq y \leq U$. Thus, the expected loss value F is derived as follows:

$$F = \int_L^U (py + q) f(y) dy + C_0 \tag{3.4}$$

The calculation process by obtaining Eqn.(3.5), please refer to the Appendix A.

$$F = +py_0^2 e^{(\mu + \frac{1}{2}\sigma^2)} \Phi(d_1) - q\Phi(d_2) + C_0 \tag{3.5}$$

where,

$$d_1 = \frac{\ln(L/y_0) - (\mu + \sigma^2)}{\sigma}, \quad d_2 = \frac{\ln(U/y_0) - \mu}{\sigma}$$

3.2. **Cash flow analysis.** With respect to a profit and loss analysis in the manufacturing industry, we have researched from a mathematical point of view in order to develop a strategy for the allocation balanced revenue by integrating both of management and production division. A corporate revenue is proportional to the production lead time and the order rates. The order rates can be regarded as a demand distribution by analyzing from the viewpoint of quantitative. Order rate was difficult to order at a constant rate throughout the year. In other words, the order rate varies every month. Considering our experience, the product value sometimes becomes to decrease in inverse proportion to the volume of orders.

A corporate management is for an investment in production business about a product volume efficiency. Therefore, we need to quantify the product size. When investing, its limit exists in the company capability for business sales. Therefore, the corporate management must determine an investment to get the product value in consideration of the demand distribution.

Consequently, it can be said that it is realistic to assume that a cash flow will be also the same log-normal distribution. Therefore, a cash flow model $S_i(t)$ is defined as follows:

Definition 3.1. *Definition of a cash flow model*

$$dS_i(t) = \mu S_i(t) dt + \sigma_i S_i(t) dW^{S_i}(t) \tag{3.6}$$

where, $S_i(0)$ is an initial plan expense that is considered to be needed at the time of manufacturing, the left-hand side is a monthly rate of return, and a rate of return varies with expected value μ_s and $\sigma_i^2 t$. Further, σ_i^2 represents variance, and $W^{S_i}(t)$ standard Brownian motion. Equation of repayment guarantee money will be described.

Now, repayment guarantee money for a loan from financial institutions can be defined by the following equation.

Definition 3.2. *H : Repayment guarantee money for a loan*

$$H = E\left[\max(P - S_i(t), 0) \cdot \frac{1}{(1+r)^i}\right] \tag{3.7}$$

Here, because $1/(1+r)^i$ of Eqn.(3.7) means that it is a case where equipment manufacturing is performed within a year, $E[\cdot]$ is an expected value under a risk neutral probability[15]. This expense H can be represented as European Put Option[15]. In other words, regarding repayment guarantee expense, from *Black · Scholes* model, an appraisal value of guarantee at the time of

each repayment can be indicated as follows[5]:

$$H = E\left[\max(P - S_i(t), 0) \cdot \frac{1}{(1+r)^i}\right]$$
$$= \frac{P}{(1+r)^i} \cdot \Phi(-h + \sigma_i \sqrt{i}) - S_i \cdot \Phi(-h) \tag{3.8}$$

where, i as it is of single-year, and h is indicated by the following equation:

$$h = \frac{\ln(S_i/P) + (r + (1/2)\sigma_i^2 i}{\sigma_i \sqrt{i}} \tag{3.9}$$

Also, $\Phi(\cdot)$ indicates a probability value of standard normal distribution, and is indicated by the following equation:

$$\Phi(h) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{h} \exp\left(-\frac{1}{2}x^2\right)dx \tag{3.10}$$

An appraisal value of equipment manufacturing will be described. If financial institutions guarantee repayment P of a small-to-midsize firm with an equipment manufacturing period, the company can avoid a default risk completely if it pays H. At that time, because all money is paid back to the debt guarantor of a company (the company president in the case of a small-to-midsize firm), the appraisal value of such credit obligation is equal to the amount of the loan. On the other hand, because asset value J of manufacturing equipment is a remaining value after subtracting a repaid money amount from a cash flow, it can be represented [5].

$$J = E\left[\max(S_i(t) - P, 0) \cdot \frac{1}{(1+r)^i}\right]$$
$$= S_i \cdot \Phi(+h) - \frac{P}{(1+r)^i} \cdot \Phi(h - \sigma_i \sqrt{i}) \tag{3.11}$$

Then, a cash flow model is rewritten (the subscript i is omitted)

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t) \tag{3.12}$$

At this time, a evaluation of the end of the year,

$$J(S,T) = E\left[\max(S(T) - P, 0)\right] \tag{3.13}$$

Therefore, A evaluation equation is [15]

$$J(S,T) = S \cdot \Phi(d_3) - P \cdot e^{-ri} \cdot \Phi(d_4) \tag{3.14}$$

where, d_3 and d_4 are

$$d_3 = \frac{\ln(S/P) + (r + (1/2)\sigma^2)i}{\sigma \sqrt{i}} \tag{3.15}$$

$$d_4 = \frac{\ln(S/P) + (r - (1/2)\sigma^2)i}{\sigma \sqrt{i}} \tag{3.16}$$
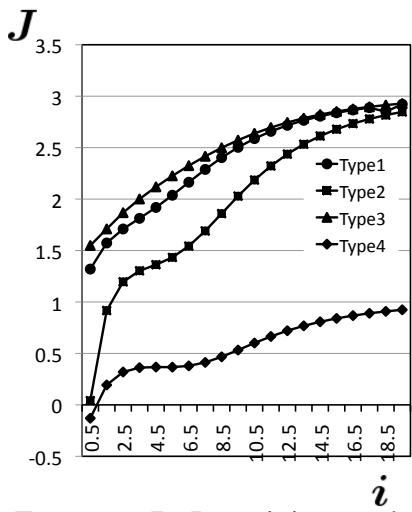
4. **Numerical Example.**

FIGURE 7. Remaining value obtained by subtracting repaid money from cash flow
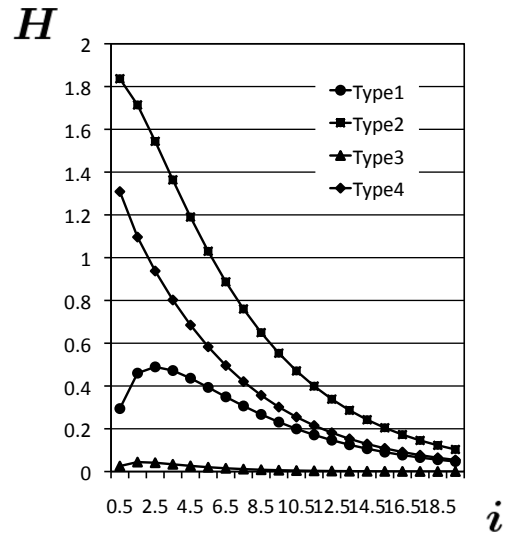
FIGURE 8. Appraisal value of guarantee at the time of repayment
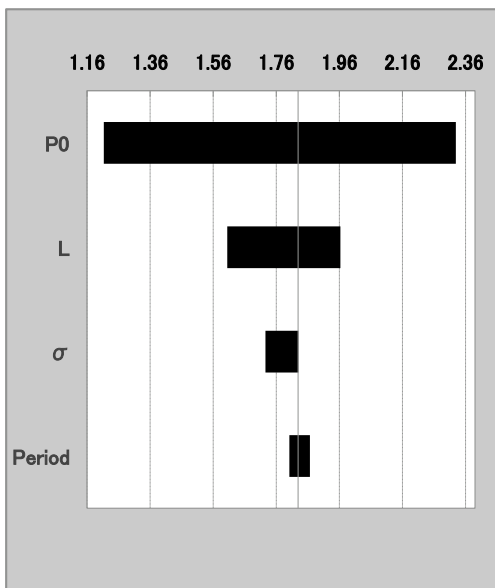
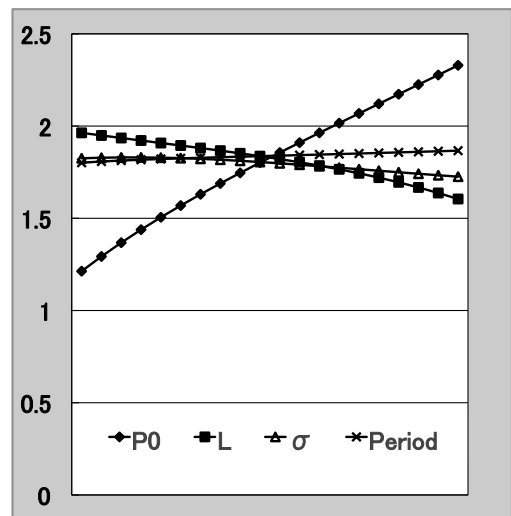FIGURE 9. Sensitivity of parameters to target value J (tornado chart)

FIGURE 10. Sensitivity of parameters to target value J (spider chart)

TABLE 1. Set parameter values

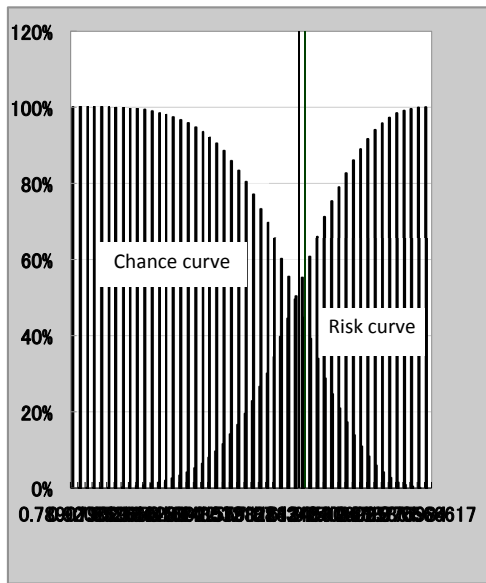|	Type 1	Type 2	Type 3	Type 4
Repayment	2.5	5	2.5	2.5
Initial value of cash flow	3	3	3	1
Volatility(σ)	0.8	0.8	0.3	0.8
Risk-free rate	0.2	0.2	0.2	0.2

FIGURE 11. Measurement of risk when each parameter is made to vary relative to target value J
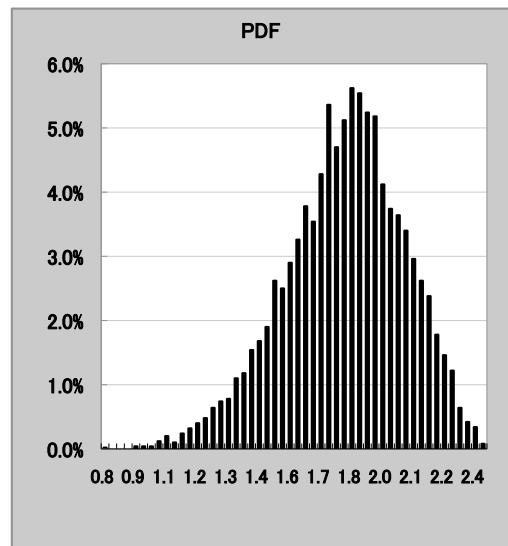

FIGURE 12. Probability distribution chart when each parameter is made to vary randomly within setting range

4.1. **Numerical simulation.** Figure 7 is results of simulation based on Eqn. (3.11). Type 1 through Type 4 in Table.1, representing the possible values when changing the parameters. Referring to Figure 7, Type 1 is a setting that is not problematic to a manufacturing plan at all. In Type 2, although a repaid money amount was set high, because a cash flow initial value is not low, a remaining value becomes high if repayment is performed early. In Type 3, even when variance had been made lowered, no effect was observed. In Type 4, if a cash flow initial value is low, as a matter of course, due to delayed repayment date, a remaining value becomes low.

Figure 8 is a diagram in which Eqn.(3.8) is graphed. Type 1 is an example in which a repayment guaranteed amount is low, a risk is relatively low, and profit is not oppressed. Type 2 is an example in which a repaid money amount is set high, and, naturally, a repaid money amount is inversely proportional to a repayment period. It can be said that a risk is high. Type 3 is an example where there is a little variation in a cash flow, and, because a repayment guaranteed amount is not high, it is an example of the lowest risk. Type 4 has the same tendency with Type 2. However, because a cash flow initial value is the lowest, and thus a repayment guaranteed amount is inversely proportional to repayment date, it can be said that a risk is high.

Sensitivity analysis and risk analysis of remaining value J is described. Figure 9 through Figure 11 show results of utilizing a simulation tool DECISION SHARE ($http = //www.integratto.co.jp/$) for quantitative evaluation.

Figure 9 is referred to as a" tornado chart", and, on the longitudinal axis, indication is made in order of a degree of influence from highest to lowest starting from the upper side. Horizontal axis shows a value that J, which is a target value, can take. This indicates that, when J is set to a target value, sensitivity of planned value P_0, repaid money amount L, standard deviation of cash flow σ and a repayment period to the target value. Figure 10 is a diagram referred to as a "spider chart", and its longitudinal axis indicates a range that the three parameters can take, and the horizontal axis indicates a value that J can take, and, in essence, it represents the same things as Figure 9. The most influential parameter is a planned value, the next is a repaid money amount followed by standard deviation, and, finally, a repayment period. Figure 11 indicates that, when target value $J \approx 2.0$, a risk is about 60 . In Figure 11, although simulation was performed with a reference value 3.0, it can be found that this target value itself has a high risk. As a target value recommended by the simulation, in the case of the parameter values set here, $J \approx 2.0$ is recommended. About whether this has a high feasibility as an execution plan or not needs to be reviewed once again in the research project. When promoting equipment manufacturing, it will be beneficial information that an uncertainty element has been made clear as above.

Figure 12 is a diagram obtained by making initial plan money amount (P_0), repayment (L), standard deviation of a cash flow (σ) and repayment period (*Period*), which have been adopted as a parameter, vary with the probability distribution. The ranges are $P_0 = 2.4 \rightarrow 3.6, L = 1.6 \rightarrow 2.4, \sigma = 0.4 \rightarrow 0.9, Period = 0.1 \rightarrow 0.4$.

4.2. **Verification of a break-even point by actual data using a production flow process.** Here, the trend coefficient, which is the ratio of actual number to the target number of pieces of equipment, represents a factor that indicates the trend for the number of pieces of production equipment, as shown in Table 2. We obtained the data of Table.2 from actual data of Test run1 through Test run3. Test run1 is an asynchronous process and Test run2/Test run3 are a synchronous processes in Appendix B.

Since the point of a break-even status is derived as $H = J$ in Eqns.(3.8) and (3.14), the inspection of Figures 13, 15, and 17 shows that the values of the guaranteed premium vary with the revenue values. In other words, as $S < P$, the risk for production business is high, and for $S > P$, the risk can be eliminated. Namely, the magnitude of the risk can be evaluated from Test run1 > Test run2 > Test run3. Moreover, Figures 14, 16, and 18 show that the same results are obtained for the evaluation of the revenue as for the guarantee premium.

TABLE 2. Parameter settings in Test run1 through Test run3

Test Type	Average μ	Volatility σ
Test run1	0.73	0.29
Test run2	0.92	0.06
Test run3	0.92	0.03

Table 3 shows the parameter settings of Figures 19 through 24. Figures 19, 21, and 23 show the numerical simulation of the B-S equation (Eqn.3.8). In Figure 19, the value of S shows large changes due to the high volatility. In other words, the magnitude of volatility is inversely proportional to the throughput. However, for the small volatility in Figures 21 and 23, the magnitudes of the solution data show only a small variation. The volatility is highest for Figure 20 and is lowest for Fig24; the magnitude of volatility is therefore inversely proportional to the
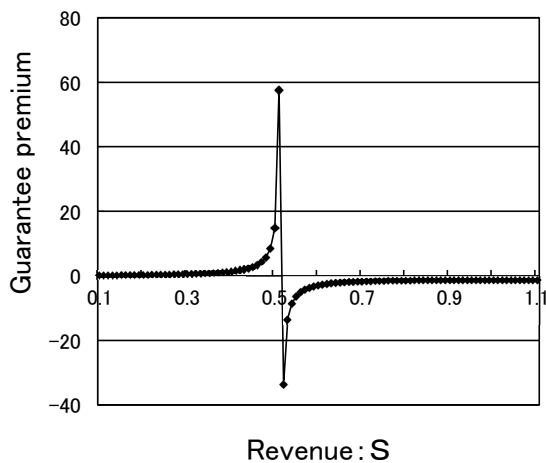
FIGURE 13. Calculation of break-even point guarantee premium in Test run1
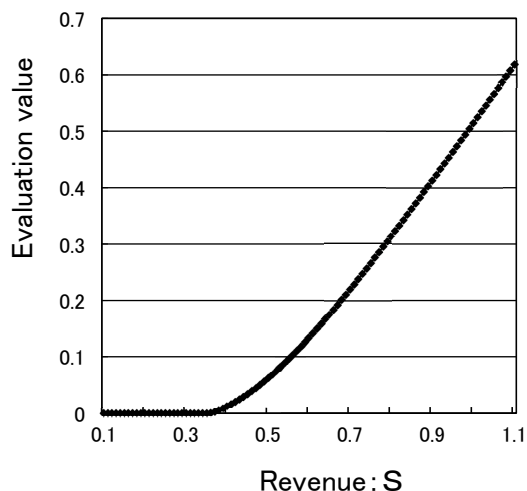

FIGURE 14. Calculation of revenue evaluation in Test run1
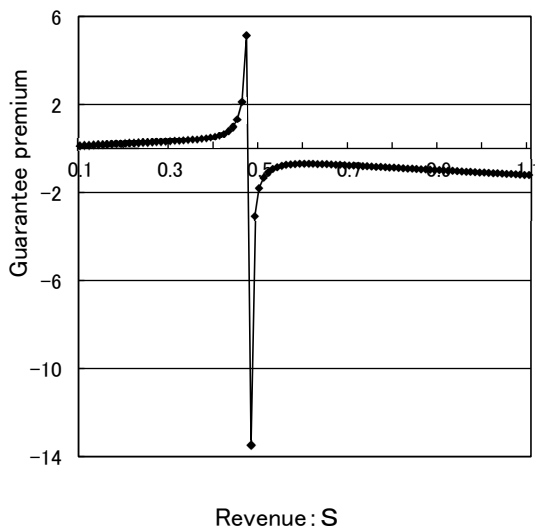

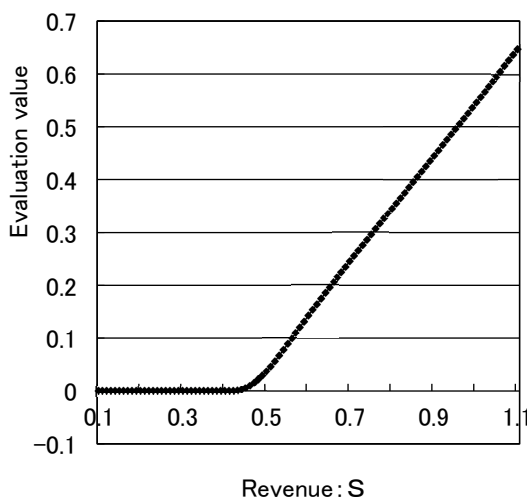FIGURE 15. Calculation of break-even point guarantee premium in Test run2

FIGURE 16. Calculation of revenue evaluation in Test run2

magnitude of the cash flow data. In other word, a small volatility indicates that the production business is stable.

4.3. **Dynamic simulation of production processes.** Regarding rate-of-return deviation, it was found that it conforms to log-normal distribution. From the analysis of mathematical models about rate-of-return deviation, we obtained the following conclusion.

First of all, analysis was made focusing attention on business rate-of-return deviation (hereinafter, referred to as rate-of-return deviation). As a result, it is reported that rate-of-return deviation has power-law characteristics[5]. Generally, disnormality of rate-of-return deviation
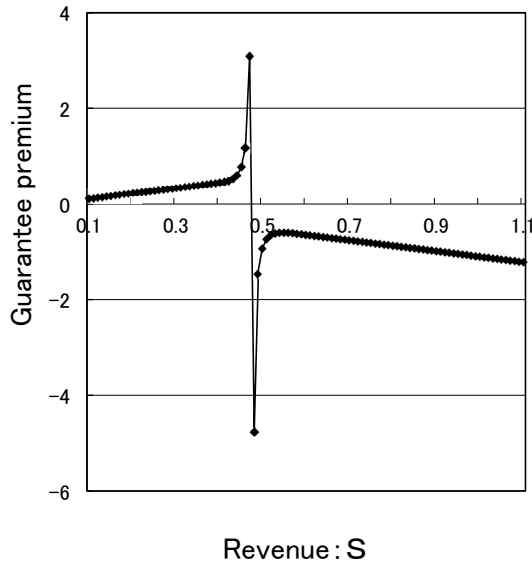
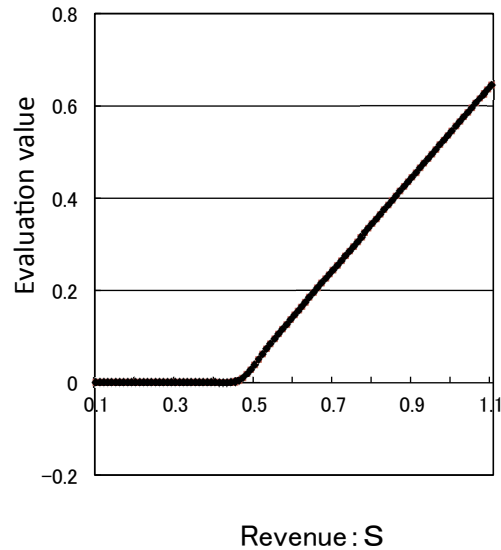FIGURE 17. Calculation of break even point guarantee premium in Test run3

FIGURE 18. Calculation of revenue evaluation in Test run3

TABLE 3. Parameter settings in Test runs 1-3

Test run	Fig	K	average μ	Volatility σ
Test run1	Figure 19	0.6	0.73	0.29
Test run1	Figure 20	0.6	0.73	0.29
Test run2	Figure 21	0.6	0.92	0.06
Test run2	Figure 22	0.6	0.92	0.06
Test run3	Figure 23	0.6	0.92	0.03
Test run3	Figure 24	0.6	0.92	0.03

in business is well known about a stock price fluctuation model, although with conditions. For example, there exists widely-known Levy process[16].

However, almost all of the reported actual data was entirely limited to stock price data. As another example, also in applying Real Option, many of the return fluctuation models are of a log-normal stochastic differential equation, and there is also one that handles a jump process[17]. Moreover, we think that, as far as the present writers and the like know, there has been no report that handles power-law distribution focusing attention on rate-of-return deviation of a privately-owned company of equipment manufacturing business. Further, regarding a make-to-order production department (production-number based manufacturing system), in relation between rate-of-return deviation and a sales amount in the case of recent production departments, a model of rate-of-return deviation becomes Langevin type. However, in reality, when a " fluctuation " becomes large, force to adjust expected values of them will be added. For example, force to adjust expected values by suppressing an order entry volume, or by making a production amount increase (or decrease) transiently will be added.

Therefore, if an amount of money of order entries and an amount of money of production are stochastic, accumulated excessive order entries becomes of Brownian motion, and thus a random " fluctuation " occurs in hour to hour order entries and production even though it might
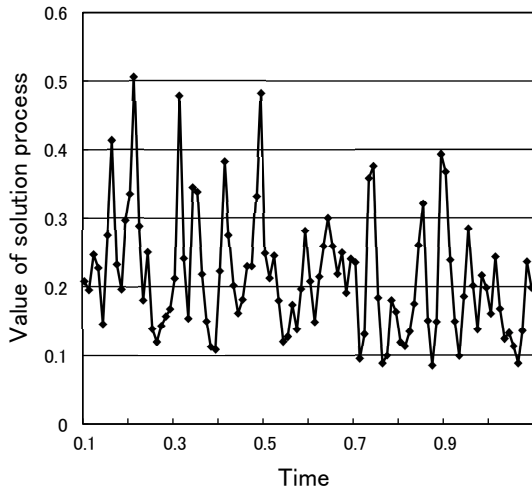
FIGURE 19. Calculation of solution processes in cash flow model Eqn.(3.6) in Test run1
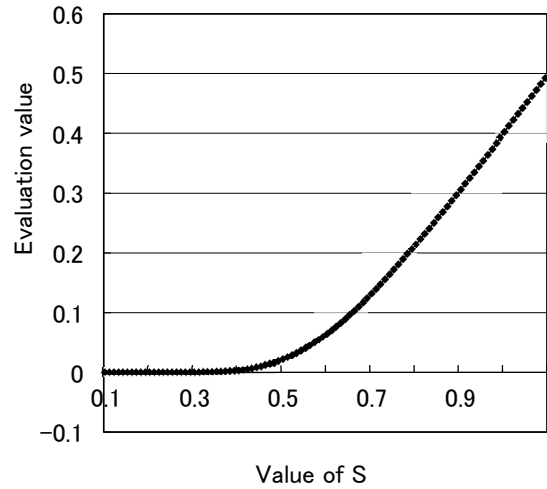
FIGURE 20. Calculation of B-S equation of Eqn.(3.14) in Test run1

FIGURE 21. Calculation of solution processes in cash flow model Eqn.(3.6) in Test run2
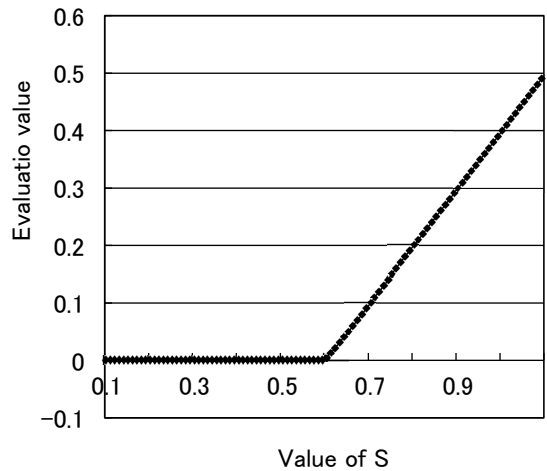
FIGURE 22. Calculation of B-S equation of Eqn.(3.14) in Test run2

be of a small degree. In addition, a rate of return is distributed log-normally, a cash flow of a target company proportional to a rate of return will be also distributed log-normally, naturally[5]. Therefore, we attempted to perform a dynamic simulation of the production process by utilizing the simulation system that NTT DATA Mathematical Systems Inc. (www.msi.co.jp) has developed. With respect to the meaning of the individual parts in Figure 25, we conducted a simulation of the following procedure.

• Individual components are commercialized by flowing from the" process1" to" process6".

FIGURE 23. Calculation of so-
lution processes in cash flow
model Eqn.(3.6) in Test run3

FIGURE 24. Calculation of B-S
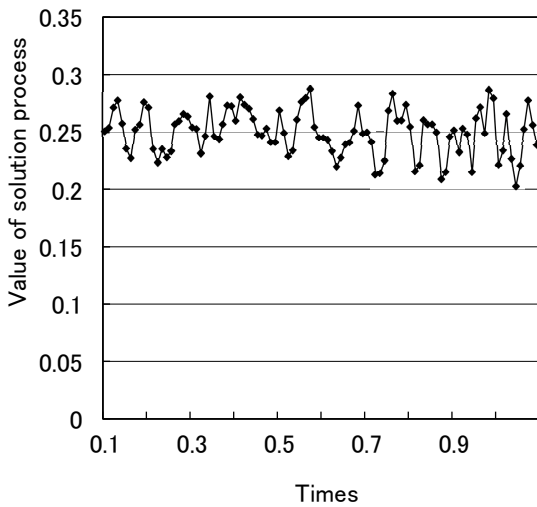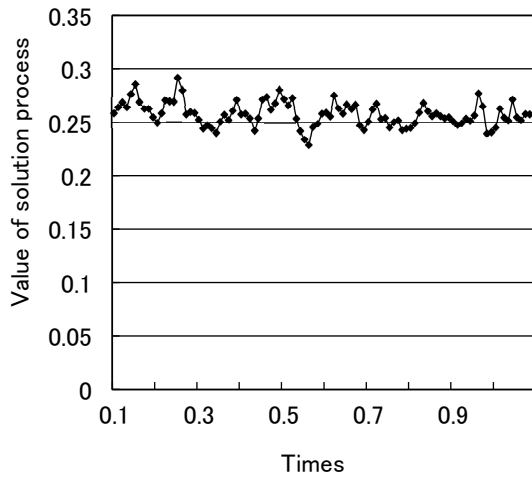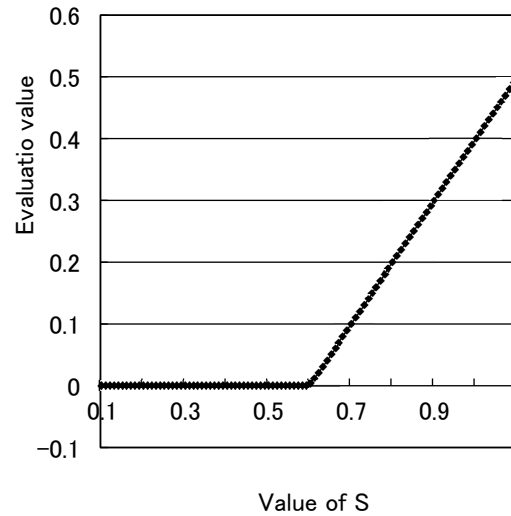equation of Eqn.(3.14) in Test
run3

- In each process, including the six workers in parallel, the slowest worker waited till the
  work was completed.
- When the work of each process was completed, it moved to the next process.
- Simultaneously as each process was completed, it recorded the working time of each pro-
  cess.

The meaning of each item in Table 4 and 5 is as follows.

- " Process No. " indicates each " process1 " to " process6 " .
- " Average " indicates the average working time.
- " STD " repersents the standard deviation (Min) at each process time of " process1 " to
  " process6 " .
- Worker's efficiency (W.E1-W.E6) represents that six workers are the ratio of the average
  working time (AVE) in each process.

" record " calculates the worker's operating time, which is obtained by multiplying the specified
WE data for the log-normally distributed random numbers in Table 4.

Figure 26 shows the operating time of process $1-6$ (record $1-$record 6). As the working
time of the synchronous process is less volatile, the work efficiency became higher than the
asynchronous process. In Figure 26, the total working time of asynchronous and synchronous
processes are 1241.7(sec) and 586.4(sec) respectively. The synchronous process shows more
better production efficiency than the asynchronous process.

5. **Conclusion.** The dynamic mathematical model, which includes the effect of procurement
of the specific production equipment from the supplier through a market, is derived and is given
by Eqn.3.6. The average and volatility are derived from the actual data Table 1 of Test run1
through Test run3. Therefore, we can construct a model for the evaluation of the production
business including the suppliers.

At present, an investment by a production business into new equipment incurs a great amount
of risk. Here, we provide a corporate strategy for the production business including the suppliers
that greatly reduces the risks due to the purchase of the new equipment. Furthermore, we
have confirmed that by expressing the cash flow model by a log-normal stochastic differential
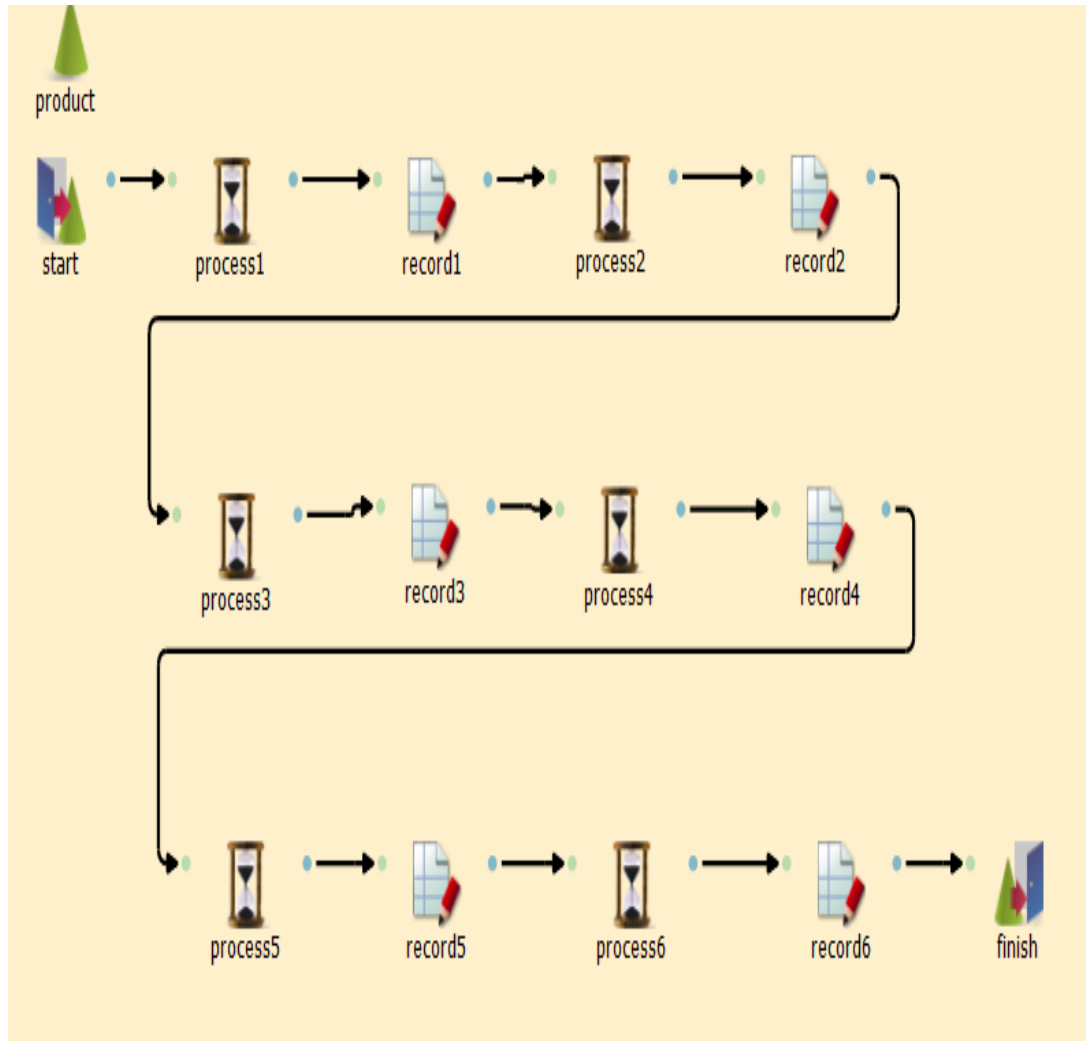
FIGURE 25. Simulation model of production flow system

TABLE 4. Working data for six production asynchronous processes

Process No.	No.1	No.2	No.3	No.4	No.5	No.6
Average	20	22	25	22	25	21
STD	2.1	2.5	1.6	1.9	2.0	1.9
W.E 1	0.83	1.0	0.66	0.76	0.88	0.91
W.E 2	1.27	1.26	1.21	1.31	1.17	1.20
W.E 3	0.96	1.11	1.01	1.12	0.88	0.89
W.E 4	0.92	0.96	1.06	0.98	0.91	0.9
W.E 5	1.2	1.03	1.07	0.89	1.03	1.1
W.E 6	1.09	1.1	1.2	0.98	1.13	0.89

TABLE 5. Working data for six production synchronous processes

Process No.	No.1	No.2	No.3	No.4	No.5	No.6
Average	20	20	20	20	20	20
STD	1.1	1.5	1.2	1.4	1.0	1.4
W.E 1	1.0	1.0	1.0	1.0	1.0	1.0
W.E 2	1.0	1.0	1.2	1.3	1.1	1.2
W.E 3	1.7	1.1	1.0	1.1	1.0	1.0
W.E 4	1.0	1.0	1.0	1.0	1.0	1.0
W.E 5	1.0	1.0	1.0	1.0	1.0	1.0
W.E 6	1.0	1.3	1.2	1.0	1.1	1.0

FIGURE 26. Working time for process number one through six

equation, the results of the research conducted in mathematical finance can also be used for the evaluation of a manufacturing company. This provides useful tools for the evaluation of a business plan.

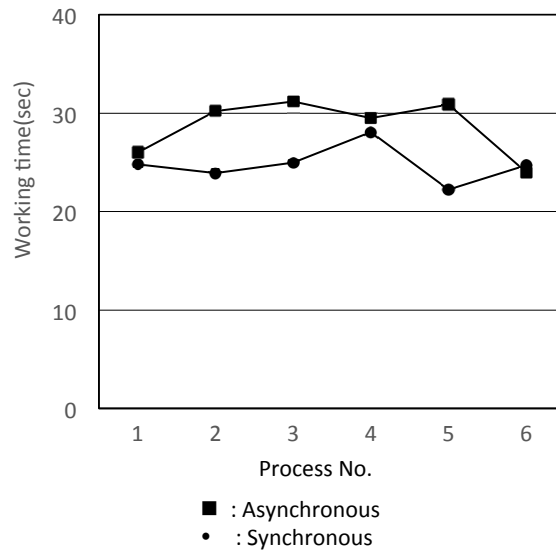Companies function in an increasingly complex and globalized financial environment, requiring greater sophistication in their financial management. We believe that the present research will make a significant contribution to the cash flow management of small-to-midsize firms.

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Appendix A. **Calculation process of obtaining Eqn.(3.5).** Therefore, the second term of Eqn.(3.4) is

$$(The\ second\ term) = \int_L^U (py+q)f(y)dy + C_0$$
$$= \int_L^\infty py \cdot f(y)dy - \int_U^\infty q \cdot f(y)dy + C_0 \tag{A.1}$$

From Eqn.(A.1), the first term of Eqn.(A.1) is

$$(The\ first\ term) = p\int_L^\infty f(y)\cdot ydy = \int_L^\infty py\cdot f(y)dy + \int_L^\infty q(U)\cdot f(y)dy \tag{A.2}$$

$$(The\ first\ term) = \int_L^\infty py\cdot f(y)dy = p\int_L^\infty \frac{1}{\sqrt{2\pi}\sigma(y/y_0)}\exp\left[-\frac{(\ln y-\ln y_0-\mu)^2}{2\sigma^2}\right]dy \tag{A.3}$$

In Eqn.(A.3), let $\ln y = x$, $y = e^x$ and then $dy = e^x dx$.

$$(The\ first\ term) = p\int_L^\infty y\cdot f(y)dy = p\int_L^\infty \frac{y_0}{\sqrt{2\pi}\sigma}\exp\left[-\frac{(x-\ln y_0-\mu)^2}{2\sigma^2}\right]e^x dx \tag{A.4}$$

Further, let $z = (x-\ln y_0-\mu)/\sigma$ and then $dx = \sigma dz$.

The first term of Eqn.(A.4) is

(*The first term*)

$$= py_0 \int_{\ln L}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}z^2\right) \exp(\sigma z + \ln y_0 + \mu) \cdot \sigma dz$$

$$= py_0 \int_{\frac{\ln L - \ln y_0 - \mu}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z^2\right) \exp(\sigma z + \ln y_0 + \mu) dz$$

$$= py_0 \int_{\frac{\ln L - \ln y_0 - \mu}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} \cdot e^{\sigma z} \cdot e^{\ln y_0 + \mu} dz = py_0^2 \int_{\frac{\ln L - \ln y_0 - \mu}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z^2 - 2\sigma z + \sigma^2) + \frac{1}{2}\sigma^2} \cdot e^{\mu} dz$$

$$= py_0^2 \int_{\frac{\ln L - \ln y_0 - \mu}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z - \sigma)^2} \cdot e^{\mu + \frac{1}{2}\sigma^2} dz = py_0^2 e^{(\mu + \frac{1}{2}\sigma^2)} \Phi\left(\frac{\ln(L/y_0) - (\mu + \sigma^2)}{\sigma}\right) \quad (A.5)$$

Applying the same method to the first term of Eqn.(A.1), the second term of Eqn.(A.1) becomes

(*The second term*)

$$= q \int_U^{\infty} f(y) dy = q \int_U^{\infty} \frac{1}{\sqrt{2\pi}\sigma(y/y_0)} \exp\left\{-\frac{(\ln y - \ln y_0 - \mu)^2}{2\sigma^2}\right\} dy \quad (A.6)$$

In Eqn.(A.6), let $\ln y = x$, $y = e^x$ and then $dy = e^x dx$.

$$(The\ second\ term) = qy_0 \int_{\ln U}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x - \ln y_0 - \mu)^2}{2\sigma^2}\right\} dx$$

$$= q \int_{\frac{\ln U - \ln y_0 - \mu}{\sigma}}^{\infty} e^{-\frac{1}{2}z^2} dz = q\Phi\left(\frac{\ln(U/y_0) - \mu}{\sigma}\right) \quad (A.7)$$

$$F = +py_0^2 e^{(\mu + \frac{1}{2}\sigma^2)} \Phi(d_1) - q\Phi(d_2) + C_0 \quad (A.8)$$

Appendix B. **Test−run1 through Test−run3 results using production flow process.** In Table.6, the circle mark represents the working delay by comparing than WS data (working standard).

TABLE 6. Total manufacturing time at each stages for each worker (Test run1)

	WS	S1	S2	S3	S4	S5	S6
K1	15	⬭20	⬭20	⬭25	⬭20	⬭20	⬭20
K2	20	22	21	22	21	19	20
K3	10	⬭20	⬭26	⬭25	⬭22	⬭22	⬭26
K4	20	17	15	19	18	16	18
K5	15	15	⬭20	18	16	15	15
K6	15	15	15	15	15	15	15
K7	15	⬭20	⬭20	⬭30	⬭20	⬭21	⬭20
K8	20	⬭29	⬭33	⬭30	⬭29	⬭32	⬭33
K9	15	14	14	15	14	14	14
Total	145	172	184	199	175	174	181

TABLE 7. Standard deviation of Table.6

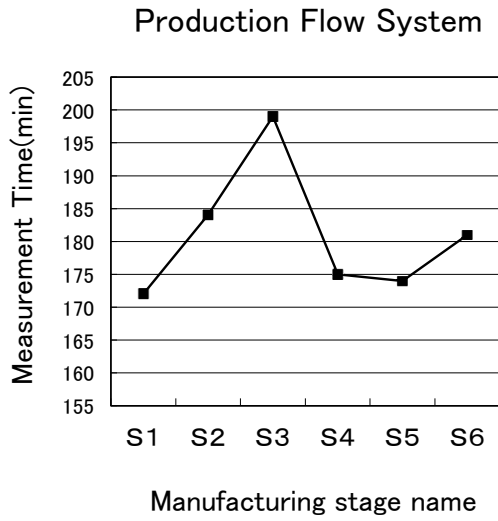| K1	1.67	1.67	3.33	1.67	1.67	1.67
K2	2.33	2	2.33	2	1.33	1.67
K3	1.67	3.67	3.33	2.33	2.33	3.67
K4	0.67	0	1.33	1	0.33	1
K5	0	1.67	1	0.33	0	0
K6	0	0	0	0	0	0
K7	1.67	1.67	5	1.67	2	1.67
K8	4.67	6	5	4.67	5.67	6
K9	0.33	0.33	0	0.33	0.33	0.33

Production Flow System

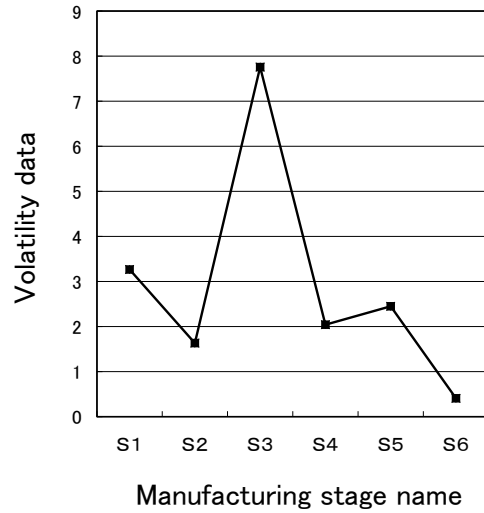FIGURE 27. Total work time for each stage(S1-S6) in Table.6

FIGURE 28. STD data for each stage(S1-S6) in Table.6

TABLE 8. Total manufacturing time at each stage for each worker (Test run2)

	WS	S1	S2	S3	S4	S5	S6
K1	20	20	24	20	20	20	20
K2	20	20	20	20	20	22	20
K3	20	20	20	20	20	20	20
K4	20	25	25	20	20	20	20
K5	20	20	20	20	20	20	20
K6	20	20	20	20	20	20	20
K7	20	20	20	20	20	20	20
K8	20	27	27	22	23	20	20
K9	20	20	20	20	20	20	20
Total	180	192	196	182	183	182	180

TABLE 9. Deviation of Table8

K1	0	1.33	0	0	0	0
K2	0	0	0	0	0.67	0
K3	0	0	0	0	0	0
K4	1.67	1.67	0	0	0	0
K5	0	0	0	0	0	0
K6	0	0	0	0	0	0
K7	0	0	0	0	0	0
K8	2.33	2.33	0.67	1	0	0
K9	0	0	0	0	0	0

TABLE 10. Total manufacturing time at each stage for each worker, K5 (*): Previous process (Test run3)

	WS	S1	S2	S3	S4	S5	S6
K1	20	18	19	18	18	18	18
K2	20	18	18	18	18	18	18
K3	20	21	21	21	21	21	21
K4	16	13	11	11	13	13	13
K5	16	*	*	*	*	*	*
K6	16	18	18	18	18	18	18
K7	16	14	14	13	14	14	13
K8	20	22	22	22	22	22	22
K9	20	20	20	20	20	20	20
Total	148	144	143	141	144	144	143

TABLE 11. Standard deviation of values stated in Table.10 K5:Previous process

K1	0.67	0.33	0.67	0.67	0.67	0.67
K2	0.67	0.67	0.67	0.67	0.67	0.67
K3	0.33	0.33	0.33	0.33	0.33	0.33
K4	1	1.67	1.67	1	1	1
K5	*	*	*	*	*	*
K6	0.67	0.67	0.67	0.67	0.67	0.67
K7	0.67	0.67	1	0.67	0.67	1
K8	0.67	0.67	0.67	0.67	0.67	0.67
K9	0	0	0	0	0	0