# Analysis of Production Processes Using a Lead Time Function


## Kenji Shirai

Department of Information System,
Niigata University of International and Information Studies,
Mizukino 3-1-1, Nishi-ku, Niigata city, Niigata Pri., Japan, 950-2292
Fax: +1-25-239-3731 E-mail: shirai@nuis.ac.jp

## Yoshinori Amano

President,
Kyonan Elecs co.LTD.,
Nishiuracho 8-48-2, Fushimi-ku, Kyoto city, Japan, 612-0029
Fax: +1-75-642-1561 E-mail: y_amano@kyohnan.co.jp


**Abstract:** We consider that production cost is included in the internal or external factors introduced by outside supplier companies (hereafter called suppliers). In this study, we analyze the changes in the lead time under this circumstance. A production business generally receives input from outside companies. Therefore, we dynamically model the specific production equipment procured from the supplier. The model is theoretically evaluated on actual return rate data, which follow a log-normal probability distribution. Furthermore, we present the actual throughput data of a production flow process with high productivity (synchronous method) and in the absence of a production flow process (asynchronous method).

**Keywords:** Lead Time Function; Production Process; Log-normal Distribution; Black-Sholes Equation.

**Biographical notes:** Kenji Shirai received his PhD in Stochastic Optimal Control System at the Ritsumeikan University. He is a Professor at the Department of Operations Research and Simulation, Niigata University of International and Information Studies. His research interests include issues related to conceptual frameworks of optimal control system, a mathematical model of production process.

Yoshinori Amano received his PhD in distributed parameters system at the Ritsumeikan University. He is a President of Kyohnan Elecs company, His research interests are related to a mathematical model and financial evaluation in production process.

## 1 Introduction

Several our previous studies have proposed financial approaches to evaluate a production business including supplier Shirai and Amano (1) (2012); Shirai and Amano (2) (2013); Shirai and Amano (3) (2014); Shirai and Amano (4) (2014).

To evaluate a production process, the lead time of production system in the production stage by using a stochastic differential equation of the log-normal type, which is derived from its dynamic behavior, is modeled Shirai and Amano (1) (2012). The use of a mathematical model that focuses on the selection process and adaptation mechanism of the production lead time is used Shirai and Amano (1) (2012). Using this model and risk-neutral integral, the evaluation equation for the compatibility condition of the production lead time is defined and then calculated. Furthermore, it is clarified that the throughput of the production process was reduced Shirai and Amano (1) (2012); Shirai and Amano (2) (2013).

With respect to determine a throughput rate, an expected value and volatility of throughput of the whole process period is estimated by utilizing Kalman filter theory having been used for a state estimation problem in the control theory Shirai and Amano (2) (2013).

With respect to a physical approach, a state in which the production density of each process corresponded to the physical propagation of heat was introduced in our previous study Shirai and Amano (6) (2012).

Using this approach, the diffusion equation, which dominates the production process was shown. Moreover, we clarified that the production process was dominated a diffusion equation Shirai and Amano (6) (2012).

To improve a production lead time, there are several studies to shorten production throughput (lead times) Sun and Hu (2010); Hu and Yue (2012). From the time of product ordering, the lead time depends on the work required to make ready for production. The several our research results which were mathematical modelings and the evaluation method of the production processes have reported.

The synchronization method is superior for improving throughput in production processes, which is used by a production flow process Shirai and Amano (5) (2013). The production flow process is utilized for production of high-mix low-volume equipments, which are produced through several stages in the production process. This method is good for producing specific control equipment such as semiconductor manufacturing equipment in our experience. Then, we have reported that the production flow process has nonlinear characteristics in our previous study Shirai and Amano (11) (2014).

Moreover, a working-time delay is propagated through the stages in the production process. Its delays are due to volatility in the model. Indeed, the actual data indicated that in the production flow process, the delays were propagated to the successive stages Shirai and Amano (6) (2012).

With respect to an actual data analysis, the rate-of-return deviation for a certain equipment manufacturer indicates a power-law distribution characteristics. Because the power-law distribution reveals the existence of a phase transition phenomenon, we expect that the rate-of-return deviation and the production system are correlated in a manner that is mediated by the power-law distribution Shirai and Amano (7) (2013). Moreover, by performing a data analysis, the relation between the rate-of-return deviation and production throughput has been clarified to some extent. A self-similar phenomenon was revealed by the fluctuation model of rate-of-return deviation and a fractal nature is shown Shirai and Amano (9) (2013); Tasaki (2000). This power-law distribution characteristic has a "fluctuating" nature during phase transition. For example, an occurrence of fluctuation is found at where the phase transition occurs at the point. The self-similarity of these fluctuations was indicated and the $f^{-1}$ and $f^{-2}$ fluctuations was also shown in our previous study Shirai and Amano (8) (2014).

Our production business utilizes the services of outside companies when ordering materials and attending to logistics. In this business environment, we analyze the changes in the lead time. For various reasons, the equipment ordered may be delayed.

To evaluate the corporate management strategy when the total production of a business depends on suppliers, we compare our model output with actual rate of return data, which follow a log-normal probability distribution. The results demonstrate the potential applicability of our proposed strategy to the manufacturing industry. We also represent actual throughput data of a company with high productivity and a company not yet adopting a production flow process. To our knowledge, we present the first analysis of lead time based on a throughput function.

## 2 Production systems in the manufacturing equipment industry

The production methods used in manufacturing equipment are briefly covered in this paper. More information is provided in our report. More information is provided in our report Shirai and Amano (7) (2013). This system is considered to be a "Make-to-order system with version control," which enables manufacturing after orders are received from clients, resulting in "volatility" according to its delivery date and lead time. In addition, there is volatility in the lead time, depending on the content of the make-to-order products (production equipment).

In Fig.1(A), the "Customer side" refers to an ordering company and "Supplier (D)" means the target company in this paper. The product manufacturer, which is the source of the ordered manufacturing equipment presents an order that takes into account the market price. In Fig.1(B), the market development department at the customer's factory receives the order through the sale contract based on the predetermined strategy.

### *2.1 Production flow process*

A manufacturing process that is termed as a production flow process is shown in Fig.2. The production flow process, which manufacture low volumes of a wide variety of products, are produced through several stages in the production process. In Fig.2, the processes consists of six stages. In each step S1−S6 of the manufacturing process, materials are being produced.

The direction of the arrows represents the direction of the production flow. production materials are supplied through the inlet and the end-product is shipped from the outletShirai and Amano (5) (2013).

In Fig.4, $T_L$ is a lower limit of lead time, and $T_U$ is a upper limit of lead time. $T$ is a lead time.

## 3 Lead time analysis using a lead time function

The lead time function $f(y)$ is assumed as a log-normal probability density function, as shown in Fig.4).

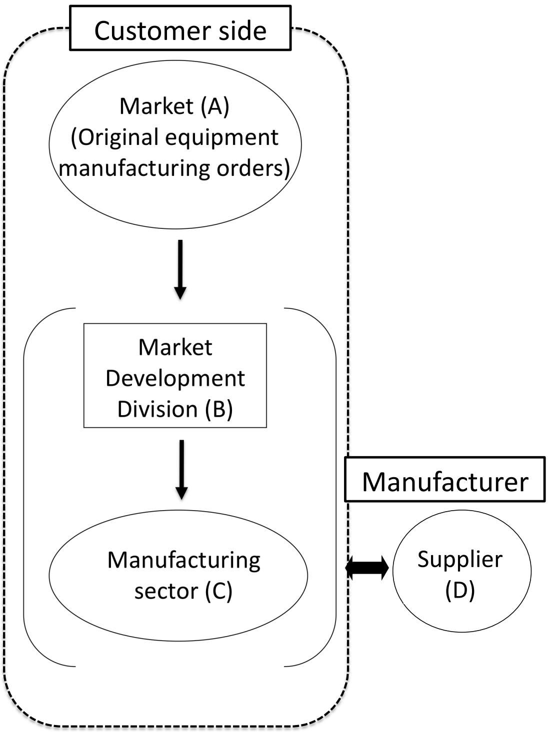


**Figure 1** Business structure of company of research target

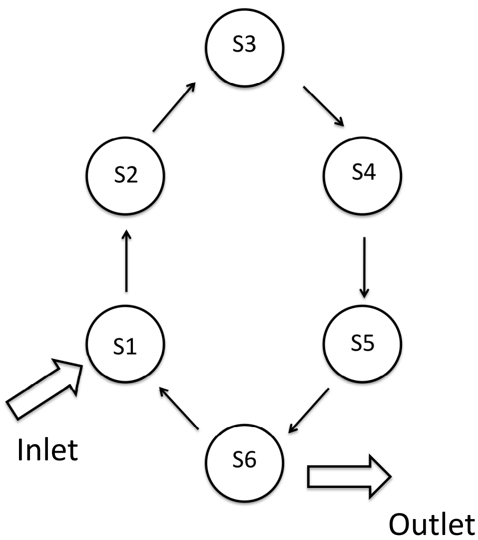


**Figure 2** Production flow process

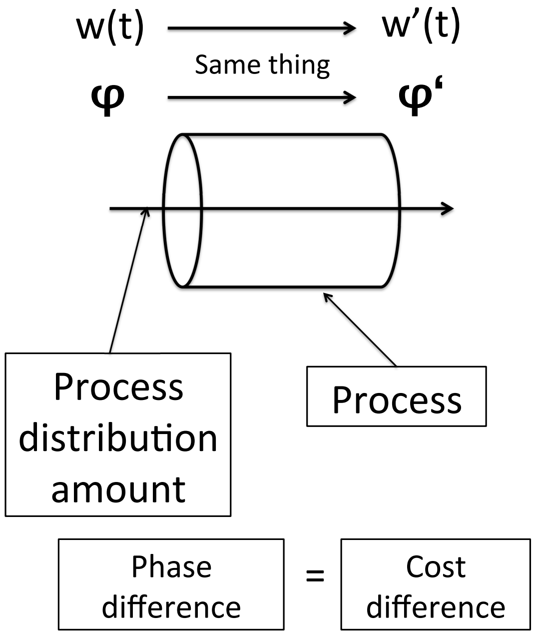


**Figure 3** Throughput fluctuation in a process distribution amount

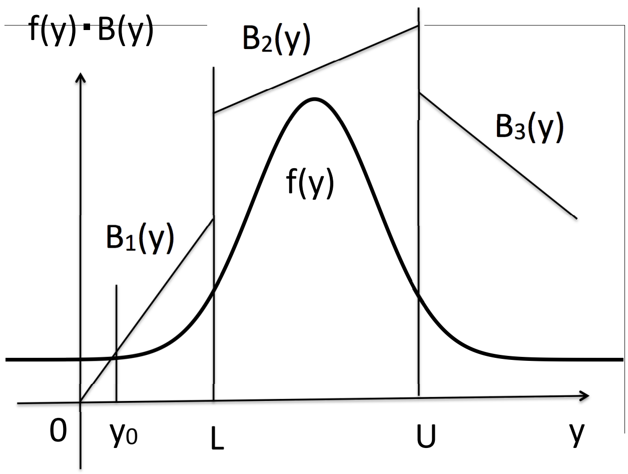


**Figure 4** Lead time function f(y) and Throughput function $B_1(y)$,$B_2(y)$ and $B_3(y)$

**Assumption 3.1:** Lead time function of a probability density function with log-normal type.

$$f(y) \equiv \frac{1}{\sqrt{2\pi}\sigma(y/y_0)} \exp\Big\{-\frac{(\ln(y/y_0)-\mu)^2}{2\sigma^2}\Big\} \tag{1}$$

where, $\mu$ is a average value, $\sigma$ is a volatility and $y_0$ is a initial lead time.

Now, let $F(L)$ as a cash-in flow and let $C_0(L)$ as a fixed cost,

$$\begin{aligned} F(L) &= \int_{-\infty}^{\infty} f(y)B(y)dy - C_0(L) \\ &= \int_{-\infty}^{L} B_1(y)f(y)dy + \int_{L}^{U} B_2(y)f(y)dy \\ &\quad + \int_{U}^{\infty} B_3(y)f(y)dy - C_0(L) \end{aligned} \tag{2}$$

where,

$$B_1(y) = p_1 y_0 + q_1(U) \tag{3}$$
$$B_2(y) = p_2 y + q_2(U) \tag{4}$$
$$B_3(y) = p_3 y + q_3(U) \tag{5}$$

Here, the relationship between a cash flow $F(L)$ and a fixed cost $C_0(L)$ at $y = L$,

$$F(L) - C_0(L) < 0 \tag{6}$$

where, $L = kU$.

The case of Eqn.(6) is impossible in practice, so requires no analysis. When $y > U$, the quantity ordered exceeds the physical limits of the production. Therefore, we must reduce the demand, and the problem becomes an analysis of $L \le U$.

In case of Eqn.(6), there is no sense in analysis. When $y > U$, it is beyond the physical limits of the production. Therefore, we can not order for such demand. Thus, the target analysis becomes $L \le U$.

$$F(U) = \int_{L}^{U} (p_2 y + q_2(U))f(y)dy \; - \; C_0(U) \tag{7}$$

Hereafter, a subscript of variable is omitted.

$$\begin{aligned} F(U) &= \int_{L}^{U} (py + q(U))f(y)dy \; - \; C_0(U) \\ &= \int_{L}^{\infty} (py + q(U))f(y)dy \\ &\quad - \int_{U}^{\infty} (py + q(U))f(y)dy \; - \; C_0(U) \end{aligned} \tag{8}$$

When $0 < L \le y \le M < \infty$ in Fig.lossfuncfig5, the throughput function is linear and given by $py + q(U)$. Thus, we obtain

$$py + q(U) \equiv \xi\sqrt{y - kU} + b \tag{9}$$

In general, the higher the lead time for a given product, the lower is the throughput.

Therefore, the second term of Eqn.(2) is

$$\begin{aligned} (The\,second\;\;term) &= \int_{L}^{\infty} (py + q(U))f(y)dy - C_0(U) \\ &= \int_{L}^{\infty} py \cdot f(y)dy + \int_{L}^{\infty} q(U)f(y)dy - C_0(U) \end{aligned} \tag{10}$$

From Eqn.(10), the first term of Eqn.(10) is

$$\begin{aligned} (The\,first\,term) &= \int_{L}^{\infty} (py + q(U))f(y)dy \\ &= p \cdot y_0 \int_{L}^{\infty} \frac{1}{\sqrt{2\pi}\sigma y} \exp\Big(-\frac{\ln y - \ln y_0 - \mu}{2\sigma^2}\Big)^2 dy \end{aligned} \tag{11}$$

In Eqn.(11), let $\ln y = x$ and then $y = e^x$.

$$\begin{aligned} (The\,first\,term) &= \int_{L}^{\infty} (py + q(U))f(y)dy \\ &= p \cdot y_0 \int_{\ln L}^{\infty} \frac{1}{\sqrt{2\pi}\sigma e^x} \exp\Big(-\frac{x - \ln y_0 - \mu}{2\sigma^2}\Big)^2 e^x \cdot (e^x \cdot dx) \end{aligned} \tag{12}$$

Further, let $z = (x - \ln y_0 - \mu)/\sigma$ and then $dx = \sigma dz$.

The first term of Eqn.(12) is

$$\begin{aligned} &(The\,first\,term) \\ &= p \cdot y_0 \int_{\ln L}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} \exp\Big(-\frac{1}{2}z^2\Big)\exp(\sigma z + \ln y_0 + \mu) \cdot \sigma dz \\ &= p \cdot y_0 \int_{\frac{\ln L - \ln y_0 - \mu}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\Big(-\frac{1}{2}z^2\Big)\exp(\sigma z + \ln y_0 + \mu)dz \\ &= p \cdot y_0 \int_{\frac{\ln L - \ln y_0 - \mu}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} \cdot e^{\sigma z} \cdot e^{\ln y_0 + \mu}dz \\ &= p \cdot y_0 \int_{\frac{\ln L - \ln y_0 - \mu}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z^2 - 2\sigma z + \sigma^2) + \frac{1}{2}\sigma^2} \cdot e^{\ln y_0 + \mu}dz \\ &= p \cdot y_0 \int_{\frac{\ln L - \ln y_0 - \mu}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z-\sigma)^2} \cdot e^{\ln y_0 + \mu + \frac{1}{2}}dz \\ &= p y_0^2 e^{(\mu + \frac{1}{2}\sigma^2)} \Phi\Big(\frac{\ln(L/y_0) - (\mu + \sigma^2)}{\sigma}\Big) \end{aligned} \tag{13}$$

Applying the same method to the first term of Eqn.(12), the second term of Eqn.(10) becomes

$$\begin{aligned} &(The\,second\,term) \\ &= q(U) \int_{L}^{\infty} f(y)dy \\ &= q(U) \cdot y_0 \int_{L}^{\infty} \frac{1}{\sqrt{2\pi}\sigma y} \exp -\Big[\frac{(\ln y - \ln y_0 - \mu)^2}{2\sigma^2}\Big]dy \end{aligned} \tag{14}$$

In Eqn.(14), let $\ln y = x$ and then $y = e^x$.

$$\begin{aligned} &(The\,second\,term) \\ &= q(U) \cdot y_0 \int_{\ln L}^{\infty}\int_{L}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} \exp -\Big[\frac{(x - \ln y_0 - \mu)^2}{2\sigma^2}\Big]dx \\ &= q(U) \cdot y_0 \int_{\frac{\ln L - \ln y_0 - \mu}{\sigma}}^{\infty} e^{-\frac{1}{2}z^2}dz \\ &= q(U) \cdot y_0^2 \Phi\Big(\frac{\ln(L/y_0) - \mu}{\sigma}\Big) \end{aligned} \tag{15}$$

The second term of Eqn.(8) is

$$(The\, second\, term)$$
$$= -\Big[py_0^2 e^{(\mu+\frac{1}{2}\sigma^2)}\Phi\Big(\frac{\ln(U/y_0)-(\mu+\sigma^2)}{\sigma}\Big)$$
$$+ q(U)\cdot y_0^2\Phi\Big(\frac{\ln(U/y_0)-\mu}{\sigma}\Big)\Big] \qquad (16)$$

From Eqs.(13) and (16),

$$F(U) = +p\cdot y_0^2 e^{(\mu+\frac{1}{2}\sigma^2)}\Big\{\Phi(d_1)+\Phi(d_2)\Big\}$$
$$- q(U)\cdot y_0^2\Big\{\Phi(d_3)+\Phi(d_4)\Big\} - C_0(U) \qquad (17)$$

where,

$$d_1 = \frac{\ln(L/y_0)-(\mu+\sigma^2)}{\sigma} \qquad (18)$$
$$d_2 = \frac{\ln(U/y_0)-(\mu+\sigma^2)}{\sigma} \qquad (19)$$
$$d_3 = \frac{\ln(L/y_0)-\mu}{\sigma} \qquad (20)$$
$$d_4 = \frac{\ln(U/y_0)-\mu}{\sigma} \qquad (21)$$

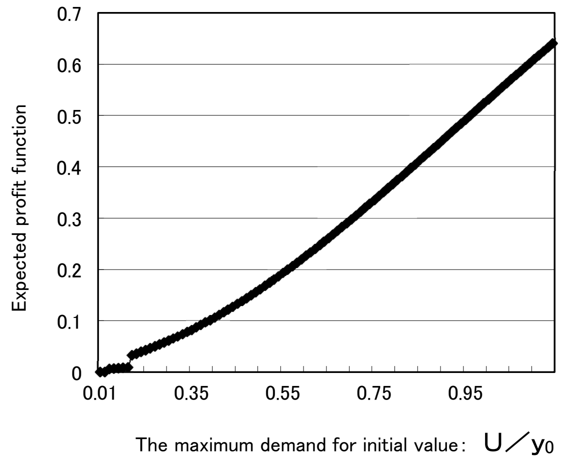


**Figure 5** Expected revenue function for demand distribution

## 4 Numerical example

Table 1 presents the parameters used to generate Figs.5 and 6, obtained by numerically calculating Eqn.(17). The parameters of Fig.5 yield a much higher expected revenue than those of Fig.6, because of the small volatility. Both Fig.5 and Fig.6 plot the expected profit $F(U)$ in Eqn.(17)for different initial maximum demands $y_0$.

Table 2 presents the parameter settings used to generate Fig.7 and 8, again obtained by numerically calculating Eqn.(17). As found above, the parameters used to generate Fig.7 yield much higher expected revenue than those of Fig.6 because of small volatility. Figs.7 and 8 plot the expected profit $F(U)$ in Eqn.(17) for different $T/t_0$. $T/t_0$, defined as the time relative to the product completion time.

**Table 1** Parameter settings in Fig.5 and Fig.6

| Figure | Average $\mu$ | Volatility $\sigma$ | $k$ | $\xi y_0$ | b |
|---|---|---|---|---|---|
| Fig.5 | 1 | 0.1 | 0.1 | 2 | 1 |
| Fig.6 | 0.7 | 0.3 | 0.5 | 1 | 1 |

**Table 2** Parameter settings in Fig.7 and Fig.8

| Figure | Average $\mu$ | Volatility $\sigma$ | $k$ | $\xi y_0$ | b |
|---|---|---|---|---|---|
| Fig.7 | 0.8 | 0.1 | 0.1 | 1 | 1 |
| Fig.8 | 0.8 | 0.3 | 0.5 | 1 | 1 |

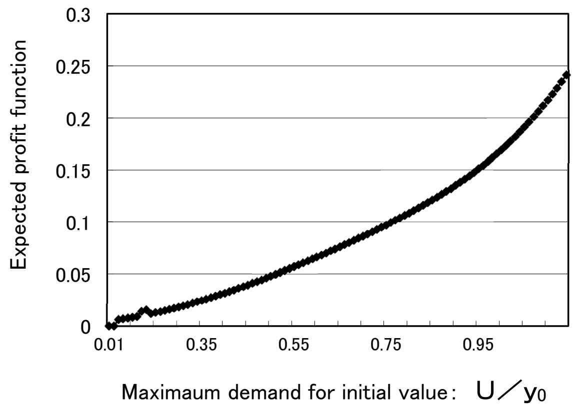


**Figure 6** Expected revenue function for demand distribution

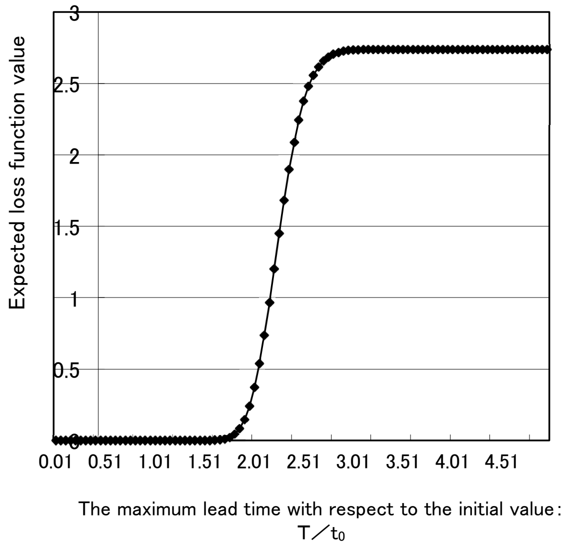


**Figure 7** The expected loss function for lead time variability distribution

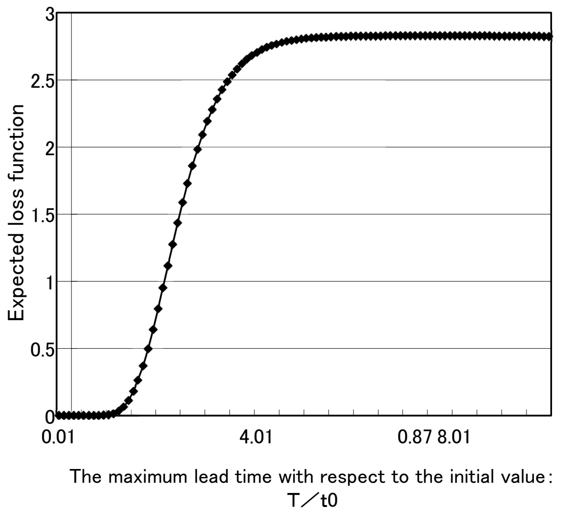


**Figure 8** The expected loss function for lead time variability distribution

## 5 Evaluation of Lead time function and Throughput function

### *5.1 Evaluation of Lead time function using a rate of return data*

Generally, a company predicts the demand of a particular product. We show that the distribution of the rate of return forms a lognormal probability distributionShirai and Amano (13) (2013). The predicted throughput is proportional to the rate of return. Therefore, it is assumed that the probability distribution of the throughput is also a lognormal distribution**?**.

About "Supplier (D)" in Fig.9, we calculated the return of 10 years from Apr., 1999 to Mar., 2008 on a month-by-month basis to calculate rate-of-return deviation. For a small-to-midsize firm, it is of the upmost importance not to cause default in a cash flow, and it is necessary for business continuity. As is the case with rate-of-return deviation described in the previous half, we also analyzed a return acquisition rate defined by Eqn.(22). The result is shown in Fig.9. From the data of monthly rate of return observed, its probability density function was calculated (Fig.9). As a result, it was found that the probability density function conforms to log-normal distribution (Fig.9, Theoretical).

Theoretical curve was calculated using EasyFit software (http://www.mathwave.com/), and as a result of Kolmogorov and Smirnov test, the observed values conformed to a log-normal type probability density function. Because, in the goodness-of-fit test of Kolmogorov-Smirnov, a null hypothesis that it is "log-normal" was not rejected with rejection rate 0.2, this data conforms to "log-normal" distribution. $P-value$ was 0.588. The parameters of a theoretical curve were: $\mu_p$ =- 0.134 (average), $\sigma_p$ = 0.0873 (standard deviation), $\gamma_p$ = -0.900. The theoretical curve is given by the following formula.

$$f(x) = \frac{1}{\sqrt{2\pi}(x-\gamma_p)\sigma_p} \times \exp\Big\{-\frac{1}{2}\Big(\frac{(\ln x-\gamma_p)-\mu}{\sigma_p}\Big)^2\Big\} \qquad (22)$$

### *5.2 Evaluation of throughput function*

Next, the throughput function in Fig.4 is evaluated on the number of equipment components/the target number of equipment. This factor represents the degree of the number of pieces of production equipment (see Appendix). The asynchronous method is prone to numerous worker fluctuations imposed by various delays. In contrast, worker fluctuations in the synchronous method are small. In terms of the production throughputs presented in the Appendix, the productivity ranking is test run 3 > test run 2 > test run 1, where test run1 is the asynchronous method and runs 2 and 3 are forms of the synchronous method.

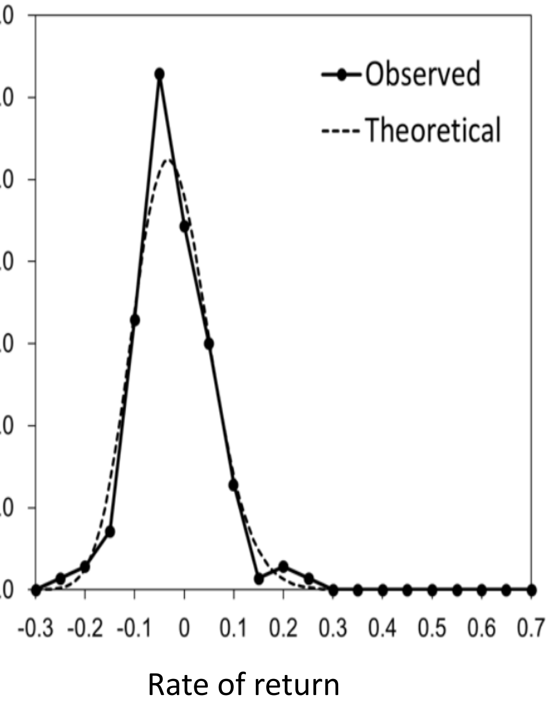


**Figure 9** Probability density function of rate-of-return deviation: actual data (solid line) and data based on theoretical formula (dotted line)

- Test-run1: 4.4 (pieces of equipment)/6 (pieces of equipment) = 0.73
- Test-run2: 5.5 (pieces of equipment)/6 (pieces of equipment) = 0.92
- Test-run3: 5.7 (pieces of equipment)/6(pieces of equipment) = 0.95

## Acknowledgment

We thank Dr. E.Chikayama, Associate professor of Niigata University of International and Information Studies, for verifying the log-normal distribution type data.

## 6 Conclusion

We theoretically analyzed the mechanism of changes in production lead time, and validated our approach on actual data. The lead time function is proportional to the rate of return, which follows a log-normal distribution. Moreover, throughput is proportional to revenue. Therefore, such analysis is essential for optimizing management in production processes.

## References

Shirai, K and Amano, Y (1). (2012) 'A Study on Mathematical Analysis of Manufacturing Lead Time -Application for Deadline Scheduling in Manufacturing System', *IEEJ Transactions on Electronics, Information and Systems*, Vol. 132-C, No. 12, pp.1973–1981

Shirai, K and Amano, Y et al (2). (2013) 'Throughput Analysis for Manufacturing Process by Using a Kalman Filter Theory under Uncertainty based on Physical Approach', *International Journal of Innovative Computing, Information and Control*, Vol. 9, No. 11, pp.4431–4445

Shirai, K and Amano, Y (3). (2014) 'Production evaluation method for improving process throughput using stochastic discount factor (I)', *International Journal of Innovative Computing, Information and Control*, Vol. 132-C, No.12, pp.1973–1981

Shirai, K and Amano, Y (4). (2014) 'Trouble analysis occurring from suppliers using option pricing theory (Ⅱ)', *International Journal of Innovative Computing, Information and Control*, Applying on September

Sun, L, Hu, X, Fang, Y and Huang, M. (2010) 'Knowledge Representation for Distribution Problem in Urban Distribution Systems', *International Journal of Innovative Computing, Information and Control*, Vol. 6, No. 9, pp.4145–4156

L.Hu, D.Yue and J.Li, (2012) 'Availability Analysis and Design Optimization for Repairable Series-Pararell System with Failure Dependencies' *International Journal of Innovative Computing, Information and Control*, Vol. 8, No. 10 (A), pp.6693–6705

Shirai, K and Amano, Y et al (5). (2013) 'Improving Throughput by Considering the Production Process' *International Journal of Innovative Computing, Information and Control*, Vol. 9, No. 12, pp.4917–4930

Shirai, K and Amano, Y (6). (2012) 'Production density diffusion equation propagation and production, *IEEJ Transactions on Electronics, Information and Systems*, Vol. 132-C, No. 6, pp.983–990

Tasaki, H, (2000) 'Thermodynamics - from a modern point of view (new physics series)' *Baifukan Co.LTD*, pp.20–80

Kitahara, K, (2000) 'Nonequilibrium statistical mechanics', *Iwanami co.,LTD*

Barada, Y and Yasuda, K. (2012) 'Testing for Lévy Type Jumps in Japanese Stock Market under the Financial Crisis using High− Frequency Data', *International Journal of Innovative Computing, Information and Control*, Vol. 8, No. 3(B), pp.2215-2223

Gerber, H, U and Shiu, E, W. (1994) 'Option Pricing by Esscher's Transforms, *Transforms of Society of Actuaries*, Vol. 46

Shirai, K, Amano, Y, et al (7). (2013) 'Power-law distribution of rate-of-return deviation and evaluation of cash flow in a control equipment manufacturing company', *International Journal of Innovative Computing, Information and Control*, Vol. 9, No. 3, pp.1095–1112

Shirai, K and Amano, Y (8). (2014) 'Self-Similarity of Fluctuations for Throughput Deviations within a Production Process', *International Journal of Innovative Computing, Information and Control*,Vol. 10, No. 3, pp.1001–1016

Shirai, K, Amano, Y et al (9). (2013) 'Consideration of Phase Transition Mechanisms during Production in Manufacturing Processes', *International Journal of Innovative Computing, Information and Control*, Vol. 9, No. 9, pp.3611–3626

Shilov, G. E and Gurevich, B. L, (1978) 'Integral, Measure, and Derivative: A Unified Approach, Richard A. Silverman', *Dover Publications*

Shirai, K and Amano, Y (10). (2014) 'Evaluation of production process using multimode vibration theory', *International Journal of Innovative Computing, Information and Control*, Vol. 10, No. 3, pp.1161–1178

Iri, M, Konno, H, and Tone, K, (1996) 'Stochastic model', *Asakura co.,LTD*

Shirai, K and Amano, Y (11). (2014) 'Throughput improvement strategy for nonlinear characteristics in the production processes', *International Journal of Innovative Computing, Information and Control*, Vol. 10, No. 3, pp.1–14

Shirai, K and Amano, Y (12). (2014) 'Application of an autonomous distributed system to the production process', *International Journal of Innovative Computing, Information and Control*, Vol. 10, No. 4, pp.1247–1265

Iryo, T and Yoshii, T, (2007)'Equivalent Optimization Problem for Finding Equilibrium in the Bottleneck Model with Departure Time Choices', *Mathematics in Transport*, pp.231–244

Shirai, K, Amano, Y et al (13). (2013) 'Process throughput analysis for manufacturing process under incomplete information based on physical approach' *International Journal of Innovative Computing, Information and Control*, Vol. 10, No. 4, pp.1247–1265

## A Appendx 1: Testing results of Test-run1 through Test-run3

Here, we represent the production method of the Test-run1 through Test-run3. Asynchronous means the delivery delay of the outside companies or means a large volatility of the workers in the production processes. A synchronous means the reverse contents of asynchronous. It's causes are a delivery delay, or large volatility of workers in the production processes. The Table 3 represents the actual data of asynchronous. The Tabel 4 and 8 represents basically a synchronous method.

- (Test-run1): Each throughput in every process (S1-S6) is asynchronous, and its process throughput is asynchronous. Table.3 represents the production time (min) in each process. The volatilities of K3 and K8 increases due to the delay of K3 and K8 in Table.4. K3 and K8 of workers in Table.3 indicate the delay propagation of working time through S1-S6 stages. Table.4 represents the volatility in each process performed by workers. Table.3 represents the target time, and the theoretical throughput is given by $3 \times 199 + 2 \times 15 = 627$(min).
  In addition, the total working time in stage S3 is 199 (min), which causes a bottleneck. Fig.11 is a graph illustrating the measurement data in Table.3, and it represents the total working time for each worker (K1-K9). The graph in Fig.12 represents the volatility data for each working time in Table.3.

- (Test-run2): Set to synchronously process the throughput.
  The target time in Table.5 is 500 (min), and the theoretical throughput (not including the synchronized idle time) is 400 (min). Table.6 represents the volatility data of each working process (S1-S6) for each worker (K1-K9).

- (Test-run3): Introducing a preprocess stage, the process throughput is performed synchronously with the reclassification of the process. The theoretical throughput (not including the synchronized idle time) is 400 (min) in Table.7. Table.8 represents the volatility data of each working process (S1-S6) for each worker (K1-K9). From this result, the idle time must be set at 100 (min). Based on the above results, the target theoretical throughput ($T_s'$) is obtained using the "synchronization-with-preprocess" method. This goal is

$$\begin{aligned} T_s \sim\ & 20 \times 6(\text{First cycle}) + 17 \times 6(\text{Second cycle}) \\ & + 20 \times 6(\text{Third cycle}) + 20(\text{Previous process}) \\ & + 8(Idol - time) = 370(min) \end{aligned} \tag{23}$$

  The full synchronous throughput in one stage (20 min.) is

$$T_s' = 3 \times 120 + 40 = 400(\text{min}) \tag{24}$$

  The throughput becomes about 10 % reduction in result. Therefore, the "synchronization-with-preprocess" method is realistic in this paper, and it is recommended the "synchronization-with-preprocess" method in the flow production system Shirai and Amano (5) (2013).
  Now, we manufactures one equipment at 3 cycle. For maintaining the throughput of 6 units / day, the production throughput is as follows.

In Table.7, the working times of the workers K4, K7 show shorter than others. However, the working time shows around target time.

Next, we manufactured one piece of equipment in three cycles. To maintain a throughput of six units/day, the production throughput must be as follows:

$$\frac{(60 \times 8 - 28)}{3} \times \frac{1}{6} \simeq 25(min) \tag{25}$$

where the throughput of the preprocess is set as 20 (min). In (25), "28" represents the throughput of the preprocess plus the idle time for synchronization. "8" is the number of processes and the total number of all processes is "8" plus the preprocess. “60” is given by 20 (min) × 3 (cycles).

Here, the preprocess represents the working until the process itself is entered. To eliminate the idle time after classification of the processes in advance, this preprocess was introduced. In Fig.10, for example, it represents the termination of the operation of step K5 during the preprocess. By making the corresponding step K5 to be the preprocess, there are eight remaining processes. When performing the 3 cycles in Fig.10, the first cycle is $\{K1, K2, K3\}$, the second cycle is $\{K4, K6, K7\}$, and the third cycle is $\{K8, K9\}$.
After completion of the third cycle, the workers start production the next product. That is, the first production process starts the first cycle. By adopting the preprocess cycle, the third cycle is adopted in a parallel process.
At this time, the theoretical throughput ($T_s$) is as follows.

Here, the preprocess is adopted in Test-run3 only.

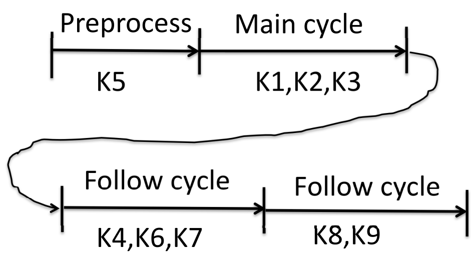


**Figure 10** “Synchronization-with-preprocess” method in production equipment

**Table 3** Total production time at each stages for each worker (Asynchronous)

| | WS | S1 | S2 | S3 | S4 | S5 | S6 |
|---|---|---|---|---|---|---|---|
| K1 | 15 | 20 | 20 | 25 | 20 | 20 | 20 |
| K2 | 20 | 22 | 21 | 22 | 21 | 19 | 20 |
| K3 | 10 | 20 | 26 | 25 | 22 | 22 | 26 |
| K4 | 20 | 17 | 15 | 19 | 18 | 16 | 18 |
| K5 | 15 | 15 | 20 | 18 | 16 | 15 | 15 |
| K6 | 15 | 15 | 15 | 15 | 15 | 15 | 15 |
| K7 | 15 | 20 | 20 | 30 | 20 | 21 | 20 |
| K8 | 20 | 29 | 33 | 30 | 29 | 32 | 33 |
| K9 | 15 | 14 | 14 | 15 | 14 | 14 | 14 |
| Total | 145 | 172 | 184 | 199 | 175 | 174 | 181 |

**Table 4** Volatility of Table3

| | | | | | | |
|---|---|---|---|---|---|---|
| K1 | 1.67 | 1.67 | 3.33 | 1.67 | 1.67 | 1.67 |
| K2 | 2.33 | 2 | 2.33 | 2 | 1.33 | 1.67 |
| K3 | 1.67 | 3.67 | 3.33 | 2.33 | 2.33 | 3.67 |
| K4 | 0.67 | 0 | 1.33 | 1 | 0.33 | 1 |
| K5 | 0 | 1.67 | 1 | 0.33 | 0 | 0 |
| K6 | 0 | 0 | 0 | 0 | 0 | 0 |
| K7 | 1.67 | 1.67 | 5 | 1.67 | 2 | 1.67 |
| K8 | 4.67 | 6 | 5 | 4.67 | 5.67 | 6 |
| K9 | 0.33 | 0.33 | 0 | 0.33 | 0.33 | 0.33 |

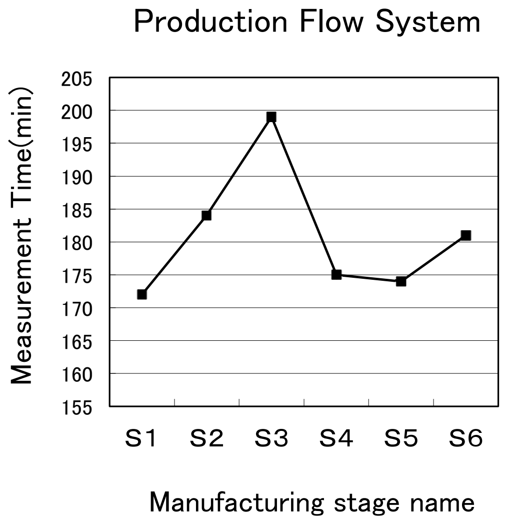


**Figure 11** Total production time of each stages by each worker

**Table 5** Total production time at each stages by each worker (Synchronous)

| | WS | S1 | S2 | S3 | S4 | S5 | S6 |
|---|---|---|---|---|---|---|---|
| K1 | 20 | 18 | 19 | 18 | 18 | 18 | 18 |
| K2 | 20 | 18 | 18 | 18 | 18 | 18 | 18 |
| K3 | 20 | 21 | 21 | 21 | 21 | 21 | 21 |
| K4 | *16 | 13 | 11 | 11 | 13 | 13 | 13 |
| K5 | *16 | 16 | 16 | 17 | 17 | 16 | 16 |
| K6 | *16 | 18 | 18 | 18 | 18 | 18 | 18 |
| K7 | 20 | 14 | 14 | 13 | 14 | 14 | 13 |
| K8 | 20 | 22 | 22 | 22 | 22 | 22 | 22 |
| K9 | 20 | 20 | 20 | 20 | 20 | 20 | 20 |
| Total | 168 | 165 | 164 | 163 | 166 | 165 | 164 |

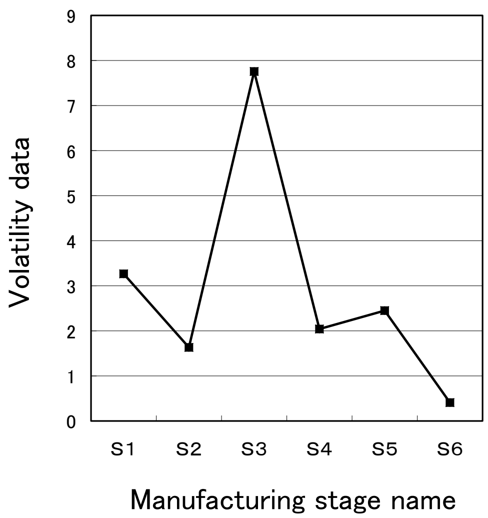


**Figure 12** STD data of each worker at each stages

**Table 6** Volatility of Table5

| | | | | | | |
|---|---|---|---|---|---|---|
| K1 | 0.67 | 0.33 | 0.67 | 0.67 | 0.67 | 0.67 |
| K2 | 0.67 | 0.67 | 0.67 | 0.67 | 0.67 | 0.67 |
| K3 | 0.33 | 0.33 | 0.33 | 0.33 | 0.33 | 0.33 |
| K4 | 1 | 1.67 | 1.67 | 1 | 1 | 1 |
| K5 | 0 | 0 | 0.33 | 0.33 | 0 | 0 |
| K6 | 0.67 | 0.67 | 0.67 | 0.67 | 0.67 | 0.67 |
| K7 | 2 | 2 | 2.33 | 2 | 2 | 2.33 |
| K8 | 0.67 | 0.67 | 0.67 | 0.67 | 0.67 | 0.67 |
| K9 | 1.67 | 1.67 | 1.67 | 1.67 | 1.67 | 1.67 |

**Table 7** Total production time at each stages for each worker (synchronous-with-preprocess), K5 (*): Preprocess

| | WS | S1 | S2 | S3 | S4 | S5 | S6 |
|---|---|---|---|---|---|---|---|
| K1 | 20 | 18 | 19 | 18 | 18 | 18 | 18 |
| K2 | 20 | 18 | 18 | 18 | 18 | 18 | 18 |
| K3 | 20 | 21 | 21 | 21 | 21 | 21 | 21 |
| K4 | 16 | 13 | 11 | 11 | 13 | 13 | 13 |
| K5 | 16 | * | * | * | * | * | * |
| K6 | 16 | 18 | 18 | 18 | 18 | 18 | 18 |
| K7 | 16 | 14 | 14 | 13 | 14 | 14 | 13 |
| K8 | 20 | 22 | 22 | 22 | 22 | 22 | 22 |
| K9 | 20 | 20 | 20 | 20 | 20 | 20 | 20 |
| Total | 148 | 144 | 143 | 141 | 144 | 144 | 143 |

**Table 8** Volatility of Table7, K5:Previous process

| | | | | | | |
|---|---|---|---|---|---|---|
| K1 | 0.67 | 0.33 | 0.67 | 0.67 | 0.67 | 0.67 |
| K2 | 0.67 | 0.67 | 0.67 | 0.67 | 0.67 | 0.67 |
| K3 | 0.33 | 0.33 | 0.33 | 0.33 | 0.33 | 0.33 |
| K4 | 1 | 1.67 | 1.67 | 1 | 1 | 1 |
| K5 | * | * | * | * | * | * |
| K6 | 0.67 | 0.67 | 0.67 | 0.67 | 0.67 | 0.67 |
| K7 | 0.67 | 0.67 | 1 | 0.67 | 0.67 | 1 |
| K8 | 0.67 | 0.67 | 0.67 | 0.67 | 0.67 | 0.67 |
| K9 | 0 | 0 | 0 | 0 | 0 | 0 |